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TEXT-BOOK OF PHYSICS.

SOUND.

BY PROFESSORS J. H. POYNTING AND J. J. THOMSON.

In Royal 8vo. Fully Illustrated.

A TEXT-BOOK OF PHYSICS.

BY

J. H. POYNTING, Sc.D., F.R.S.,
Late Fellow of Trinity College, Cambridge; Professor of Physics,
Mason University College, Birmingham;

AND

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A

TEXT-BOOK OF PHYSICS.

BY

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PREFACE.

The following account of the phenomena of Sound and of the theory connecting them together forms one part only of a Text-Book of Physics which the authors are preparing. The Text-Book is intended chiefly for the use of students who lay most stress on the study of the experimental part of Physics, and who have not yet reached the stage at which the reading of advanced treatises on special subjects is desirable. To bring the subject within the compass thus prescribed, an account is given only of phenomena which are of special importance, or which appear to throw light on other branches of Physics, and the mathematical methods adopted are very elementary. The student who possesses a knowledge of advanced mathematical methods, and who knows how to use them, will, no doubt, be able to work out and remember most easily a theory which uses such methods. But at present a large number of earnest students of Physics are not so equipped, and the authors aim at giving an account of the subject which will be useful to students of this class.

Even for the reader who is mathematically trained, there is some advantage in the study of elementary methods, compensating for their cumbrous form. They bring before us more evidently the points at which the various assumptions are made, and they render more prominent the conditions under which the theory holds

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good. For such readers the authors hope that this work may afford a fitting introduction to more advanced treatises, and especially to Helmholtz's great work, *The Sensations of Tone*, which deals chiefly with the physiological aspect of Sound, and to Lord Rayleigh's *Theory of Sound*, at once the most systematic, original, and complete work on the subject.

January 1899.

PREFACE TO SECOND EDITION.

It is with much satisfaction that we find a new edition of this work is called for within about twelve months of the issue of the first large edition. Opportunity has been taken to make a few minor corrections.

April 1900.

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SOUND.

CHAPTER I.

THE NATURE OF SOUND AND ITS CHIEF CHARACTERISTICS.

CONTENTS.—Origin of Sound and Sound-Waves—Characteristics of Waves—Characteristics of Sounds—Loudness—Pitch—Quality—Sound-Waves are Longitudinal—Noise.

THE sensation which we receive through the ear, and the external disturbance which arouses that sensation when it reaches the ear, are both denoted in common language as sound. In physics we are concerned with the external disturbance, and our aim is to investigate the conditions under which it arises, the mode in which it is propagated from its source, and the variations in the nature of the disturbance which correspond to the differences in the sensations we experience.

Sound arises in general from Vibrating Sources.—An examination of the mode of sounding any musical instrument in which the sounding parts consist of strings, rods, or bars, shows that these are always struck or pulled in such a way that they are set in vibration. Usually the vibration is so frequent that we cannot follow the motion to and fro, though it may be so wide in extent that the outline of the vibrating body is visibly altered. More generally, when any body, whether designed as a musical instrument or not, is set in vibration, if the vibrations are regular and of sufficient frequency and extent, a musical note is heard. Instances, such as a tumbler or a gas globe, will occur to the reader.

A Vibrating Source gives rise to Waves in the surrounding Air.—Let us imagine a round, flat, metal plate hung up by strings (Fig. 1, *a*) passing through holes bored at a suitable distance—about 0·68 of the radius is the best—from the centre. When struck at the centre, it vibrates between the positions shown by the dotted lines in Fig. 1, *b*, the central part always moving to and fro in the opposite direction to the rim, and the circle through the suspending holes being at rest. Considering only the central portion, we can see how its vibrations will affect the air. As the plate moves forward it pushes against the layer of air in front of it, compresses it, and drives it against the next layer. This next layer is then compressed and pushed forward against the next, and so on, the push being transmitted on through the air. When the plate has reached the limit of its forward excursion it returns, and the air follows it, expanding into the space vacated by the plate. This relief of pressure has the same effect as a pull, and therefore we may think of the plate as now pulling at the layer of air in front of it. As this layer moves backwards it lessens the pressure against the next layer, which also moves backwards, and so on. Thus, the extension or pull of the air is transmitted forwards after the push. When the plate makes another forward motion, another push is sent out, to be followed, when the plate returns, by another pull. These pushes and pulls, or compressions and extensions, with the necessary to-and-fro motions, constitute air-waves, each complete wave consisting of a push and a pull. To obtain a clear notion of the movement of the layers of air during the propagation of the waves, the reader should construct a Crova's disc like that represented in Fig. 2. Eight or ten equidistant points are taken on a small circle at the centre of a large card, and from these points, taken as centres in regular succession, circles are drawn, with radii increasing successively by a constant amount greater than the distance between the successive centres. Fifteen or twenty circles may be thus drawn, and if the card is rotated about its centre, the motion of the curved lines on one side of the centre will represent the to-and-fro motion of the layers of air, as this would appear if we could suppose the air visible and watched by an observer on one side of the line of

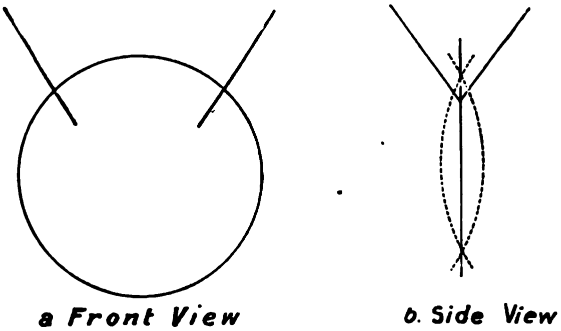

FIG. 1.—*a*. Vibrating Plate hung up by strings through points 0·68 radius from the centre.
b. Dotted lines showing the mode of vibration when struck at the centre.

propagation. If the circles are drawn on a small scale and nearly up to the centre, as in Fig. 2, it is better to cover up the card with a second fixed card having a slit cut in it, say of the width shown by the dotted lines, so that the curved lines are only seen through this slit and their curvature is less evident. But with a large disc this is quite needless.

We shall now briefly consider the evidence which leads us to believe that the waves arising from the vibrating source constitute sound.

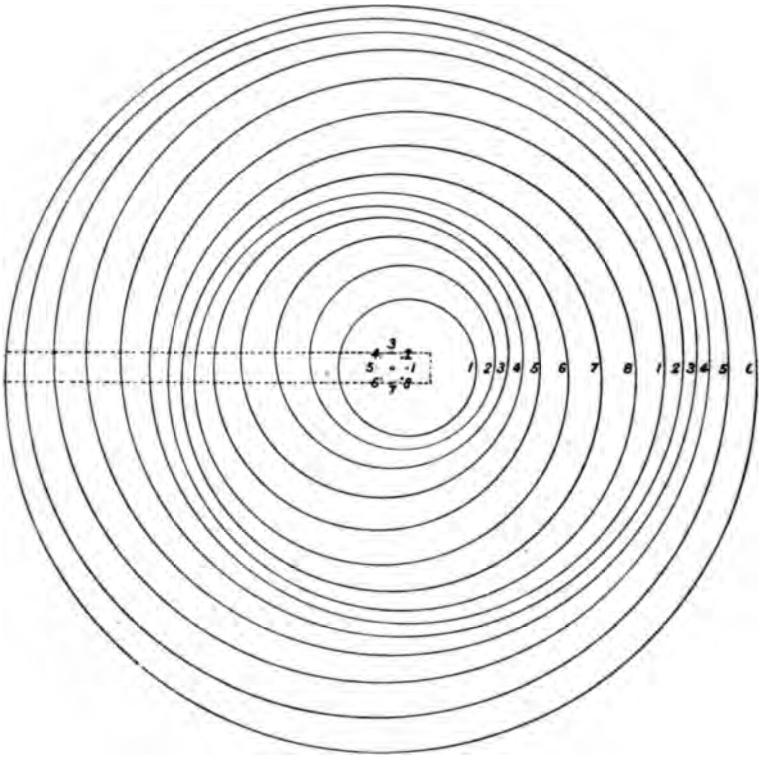

FIG. 2.—Crova's Disc. Radii of successive circles increasing by 3 mm.; drawn from 8 points round a circle of 2 mm. radius.

The Characteristics of Waves.—The characteristics of the transmission of disturbance in the form of waves are:—

1. The disturbance takes time to travel from one point to another.

2. A material medium to be disturbed is necessary.

3. On meeting an obstacle the waves are reflected back, and the angles of incidence and reflection are equal.

4. The course of the waves is changed, *i.e.* they are refracted

when they pass from one medium into another in which the rate of travel is different.

5. There are a series of phenomena grouped together under the terms **Interference** and **Diffraction** which can be shown to result from the alternating nature of the disturbance brought to any point by the waves.

The waves with which we are most familiar are those on the surface of water, and we may easily observe in them all the characteristics just described. It is obviously true that they take time to travel, and that the disturbance is in the medium—the water—through which they travel. The reflection of water-waves may be well seen on a reservoir when the waves come up against a straight wall. A little attention may be needed to separate the reflected from the incident waves, but with practice the two series will be seen quite distinctly travelling at equal inclinations on the two sides of the normal.

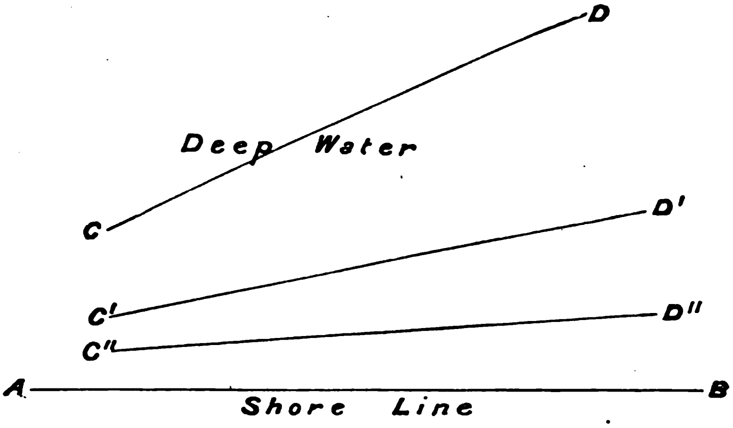

FIG. 3.—Refraction of Waves on a Sloping Shore.

Refraction occurs whenever water-waves travel obliquely towards a gradually sloping shore. The waves as they roll in become more and more nearly parallel to the shore. This may be observed on a large scale in the waves of the sea breaking on a sloping sandy shore, and on a small scale in the ripples on a pool. The explanation is to be found in the fact that as the depth decreases the rate of travel also decreases. For let AB, Fig. 3, represent the margin of the water surface, the depth gradually increasing outwards. Let CD represent a wave in deep water at a certain instant, and let the depth at C begin to diminish so as to lessen the speed of the wave. The part of the wave towards D is still in deep water, and so its speed is unchanged. A short time later, when C has reached C′, D has travelled a greater distance to D′, and so the wave has swung round. D′ now begins to travel more slowly, but C′, being in shallower water, travels more slowly still, so that while D′ travels to D″, C′ only travels to C″, and the wave swings round still more, and ultimately it may come in practically parallel to the shore-line AB.

Examples of interference of water-waves are not quite so easily detected, but one case may sometimes be seen when reflection is occurring at oblique incidence against a vertical wall. The water near the wall has a peculiar lumpy appearance, the lumps rising and falling several times in succession at nearly the same points. This appearance may be explained by the superposition and interference with each other of the incident and reflected waves.

It is easy and very interesting to study some of the characteristics of waves in the case of ripples in a shallow water-trough. Instead of looking at the ripples directly, we may illuminate the surface by a bright light—sunlight is most effective—and watch the ripples in the reflection on a suitably placed screen. Reflection may be obtained from bodies of variously shaped edges placed in the trough, and refraction may be shown by placing at the bottom of the trough bodies of lenticular or other shapes, reducing the depth from, say, 2 inches to ½ inch, the velocity of ripple-propagation being reduced at the same time.

Turning now to sound, we shall see that all the general characteristics of waves may be recognised as—

Characteristics of Sound.—1. *Sound takes time to travel.*—We can often detect an interval between the sight and sound of some event. We see the puff of smoke from a gun before we hear the report if the gun is at a distance, or we see the steam rising from the whistle of an engine before we hear the sound, and the sound continues after the steam is shut off. Still more striking is the interval between the flash of lightning and the thunder to which the lightning gives rise. In the next chapter we shall see that direct measures of the velocity of sound show that for a given state of the air it is constant, and that this constant value is the same as the velocity with which air-waves should travel as calculated on mechanical principles.

2. *Sound travels through air or some material medium.*—If an electric bell is suspended within the receiver of an air-pump and set ringing, the sound gradually decreases as the air is exhausted, and finally becomes inaudible, the material connection between the bell and the observer being insufficient to transmit a sensible disturbance. On re-admitting the air, the bell is heard again.

But sound may travel through media other than air. A watch placed on one end of a table may be heard ticking most distinctly on placing the ear against the other end of the table, the sound travelling through the wood of the table; or if the observer holds the watch in his teeth, he hears the ticking loudly through the teeth and the bones of the skull. The drum telephone, which consists of a couple of drum-heads with a tight string or wire connecting the centres of the two drums, is another instance of sound-propagation through a solid medium. The sound, passing through the air, strikes one drum, is conveyed from it along the string, and is finally yielded up to the air by the other drum.

Sound may easily be heard through water. If, for instance, the head is placed under water in a bath, the sound made by striking the side of the bath is perfectly audible.

3. *Sound is reflected.*—An echo is simply a case of sound-reflection, and whenever we hear one we find some surface to which we may ascribe the reflection. On a small scale we may reflect sound by holding a watch in front of a large concave mirror. In accordance with the law of equality of the angles of incidence and reflection, the light from a source, S, Fig. 4, is gathered together after reflection at or near another point, S′. If the watch is placed at S, its ticking is heard most loudly when the ear is placed at S′, or, better still, when a funnel with a tube leading to the ear is placed at S′. If the watch is placed at F, the principal focus of the mirror, whence light would be reflected as a parallel beam, the sound of the ticking may be thrown across a large room.

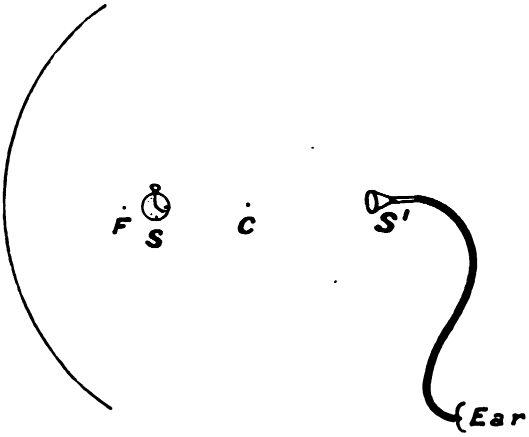

FIG. 4.—Reflection of Sound by a Concave Mirror. S, S′, conjugate foci. F, principal focus.

4. *Sound is refracted when passing from one medium into another in which its speed is different.*—Sondhauss devised a sound lens consisting of a collodion balloon of lenticular shape filled with carbonic acid. Since sound travels more slowly in this gas than in air, the sound proceeding from a point at some distance on one side of the balloon and on its axis should be concentrated at a point at a certain distance on the axis on the other side, just as light would be concentrated by a glass lens, and Sondhauss verified the existence of this concentration. Fig. 5 represents the mode in

which this is explained on the supposition that the sound consists of waves. The waves diverge from the source S, and on entering the lens travel more slowly. But as the central parts reach the lens first, they are delayed more than the edges, and therefore the waves become flattened. On emergence they travel more quickly again, and the edges emerging first gain on the central part, and so give the waves an opposite curvature which concentrates them on the point S′.

Tyndall[1] describes a very interesting experiment, in which the lens is a soap-bubble filled with nitrous oxide, the source a concertina reed, and the receiver a sensitive flame.

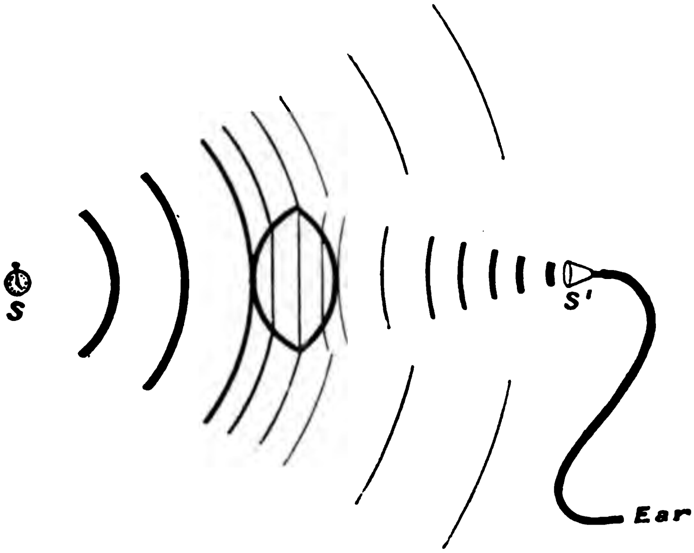

FIG. 5.—Waves Concentrated by a Lens, in which they travel more slowly. The thickness of the lines roughly represents the intensity of the waves. Radius of lens, 10; refractive index, 1·27; $SS' = 4f = 74$.

Other phenomena easily explained as cases of sound refraction will be described in Chapter II.

5. *Sound exhibits interference.*—We shall here merely mention one illustration of this, the well-known "beating" of two notes nearly, but not quite, in unison, leaving further description till we discuss the subject more fully in Chapter X.

Since, then, sound arises from vibrating bodies which must give rise to waves in the surrounding matter, and since sound exhibits all the characteristics of waves, we can come only to the conclusion that **sound consists of waves.**

The Three Characteristics of our Sensations of Sound

---

[1] *Sound*, 4th ed. p. 265.

and their Physical Correspondents.—Our sound sensations differ from one another in the three respects of **LOUDNESS**, **PITCH**, and **QUALITY**, and in no other respects. We may, without any detailed examination, assign to each of these its physical correspondent.

Loudness.—When sound is arising from a given source, both observation and experiment show that the greater the extent of vibration, and the greater, therefore, the issuing waves, the louder is the sound. The waves are not longer, but larger in the sense that the to-and-fro motion of the transmitting particles of the medium is greater. Half the length of the total swing of a particle, as a wave passes through it, is termed its **amplitude**; hence we may say that the loudness of a sound depends upon and increases with the amplitude of the waves to which it corresponds.

Up to the present time no simple and satisfactory method of measuring loudness has been devised, since it has never been of practical importance to obtain a measure. We pay much more attention to the quality than to the loudness of a given sound. If, for instance, we can hear a speaker perfectly clearly and without effort, we never think of the loudness of his voice, unless it becomes painfully loud. Indeed, there is no doubt that when we do seek to compare loudness in different cases, we make quite false judgments. Most people suppose that a peal of thunder from lightning within a mile or so is exceedingly loud; yet it may be entirely lost in the sound of street traffic in a town.

Probably if sources of sound like fog-sirens, in which loudness is the chief aim, should come into common use, modes of measuring loudness would be speedily devised, corresponding to the modes of measuring the brightness of sources of light.

Pitch.—By difference of pitch we mean the difference which exists between the different notes, say a treble and a bass note, on the same musical instrument. The former is described as higher in pitch than the latter. Many sounds have no definite pitch. The report of a gun, a peal of thunder, the roar of street traffic, are merely noises in which we may sometimes have some slight sense of higher or lower, but yet we cannot describe them by any particular note. Every musical sound, however, has a definite pitch, and we may have the same note produced in a variety of ways, as by the voice, by a piano-string, by the air in an organ-pipe, by the finger-nail run over a piece of ribbed silk, or even by a jet of water falling on a stone. In all these cases there is some source sending out regular waves, and it is easy to show by experiment that the pitch of the note heard corresponds to the number of waves reaching the ear per second, that is, to the **frequency** of the sound, the same pitch always corresponding to the same frequency, whatever the source. An apparatus devised for this purpose, a modification of the **Cardboard Siren**, is shown in Fig. 6.

D is a cardboard or metal disc mounted on a spindle, and provided with, say, four concentric rows of equidistant holes. On the same spindle are four toothed wheels, W, one for each circle of holes, and having as many teeth as there are holes in its corresponding circle. The spindle can be rapidly rotated by a belt or cord gearing it with a large wheel, which may be turned by hand. A stream of air is blown from a nozzle, T, against one of the circles of holes, and every time a hole passes under the nozzle, a puff of air issues through the card and gives rise to a wave. If the disc is uniformly turned, the waves are regular and a definite note is heard. If a thin card, C, is lightly pressed against the corresponding toothed wheel, it is lifted up and let fall by each tooth in turn, and so gives rise to waves the same in frequency as the disc, and **the note heard is the same in pitch**, however different in quality. If the speed of rotation is varied as it rises, the note rises with it, showing that higher pitch accompanies greater frequency.

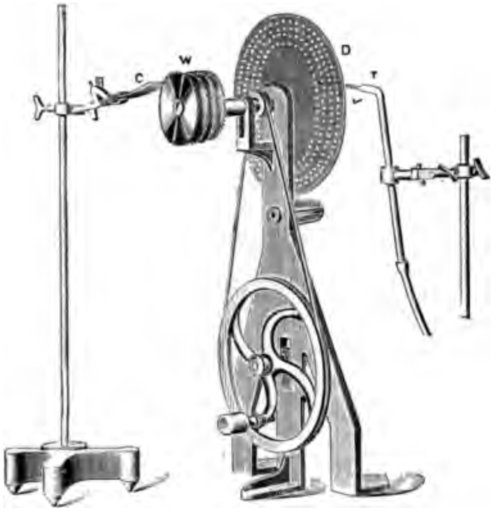

Fig. 6.—Cardboard Siren and Toothed Wheels, to show that pitch depends on frequency. W, wheels respectively having teeth equal in number to holes in circles of disc D ; C, card held against teeth ; T, tube through which air is blown against the holes.

If there are four rows of holes and four wheels, it is usual to make the numbers in the ratios 4 : 5 : 6 : 8. The last has, therefore, for a given speed, always double the frequency of the first. It is found that it always gives the octave at the same speed, whatever this may be ; or if one note has double the frequency of another, it is the octave of that other.

The notes given by the first and second rows always form the interval of a major third, and those of the first and third that of a fifth. These are special cases of the general truth that the frequencies of the two notes forming a given musical interval always bear the same ratio to each other from whatever part of the scale they are taken.

Quality.—The quality of a sound is that which distinguishes it from another sound of the same pitch proceeding from a different source. We may hear the same note sung and sounded on the piano, but there is no possibility of mistaking the one sound for the other, so entirely different is their quality. There is no doubt that the difference in quality corresponds to a difference in the form of the waves reaching the ear. We shall now consider how the waves may have different forms while still of the same length and frequency.

There are two types of simple waves, in which the disturbances or alternating motions of the particles are respectively perpendicular to and in the direction of the line in which the waves are travelling. Let waves be passing from A to B, Fig. 7, and let L_1 L_2 L_3 . . . be sections of succesive layers of the transmitting medium. Then in the first type the motion of these layers will be up and down or sideways, transverse to the direction AB, and the waves are termed **transverse waves.** In the second type, each layer moves to and fro parallel to A B and the waves are termed **longitudinal waves.** An example of a transverse wave is given by a long cord or rope fixed at one end and held in the hand at the other. On moving the hand sharply, transversely to the rope, a wave is transmitted along it, disturbing each particle transversely.

Waves on the surface of water are compounds of transverse and longitudinal motion, for a floating particle both rises up and down and moves to and fro. If the waves are made in a shallow trough in which there are particles of dust at the bottom, these particles will be seen to move to and fro, showing that at the bottom the waves are longitudinal.

A study of the mechanism of wave-transmission shows us that **Sound-Waves in Air are Longitudinal.**—For suppose that the layer of air L_1, Fig. 7, is moving up and down transversely to AB. Consider its upward motion. It only tends to move L_2 up while it is **moving past it**, the only force exerted being that of viscosity, which gradually brings L_1 and L_2 relatively to rest and then ceases. While L_1 shares in this way its momentum with L_2 and so transmits some energy to it, there is, on the whole, a loss of energy, for the transmission from a quickly to a slowly moving body implies such loss. We may compare this with the case of impact with co-efficient of restitution zero, where the two bodies only act on each other till they have a common velocity, and where there is always loss of energy. The layer L_2 will tend to move L_3 up and transmit momentum to it, but again with a loss of energy, and so on. When L_1 moves down again, it tends to drag the succeeding layers down; but there is again loss of energy in the transmission of the momentum, and the loss is so great, that though a transverse wave may be started in air, the disturbance will not penetrate very far. There is not the slightest experimental evidence for a rapid dissipation of energy in sound-waves, so that we must exclude the supposition that they are transverse.

Now suppose that L_1 is moving to and fro parallel to AB. As it moves forward against L_2 it transmits momentum, and therefore energy, to it; and as long as L_1 is moving more rapidly forward than L_2, there is a diminution of kinetic energy in the process. This is now not dissipated, but transformed to the potential energy of compression; and when L_1 and L_2 have come relatively to rest, there is the increased pressure between the two ready to produce relative motion and restore the kinetic energy in the return journey. L_2 moves forward against L_3 and shares momentum and energy with it, and so on. Then when L_1 moves back, L_2 follows it, L_3 follows L_2, and so on; and a wave of to-and-fro motion is propagated onwards.

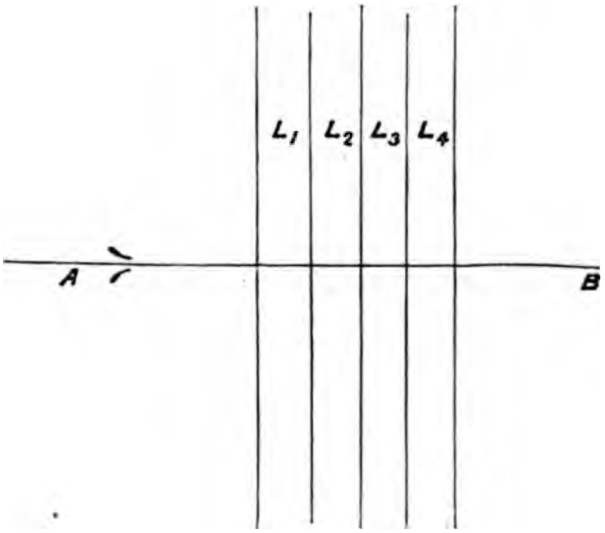

FIG. 7.

It may be pointed out that in a solid medium transverse waves can be propagated without loss of energy, for the kinetic energy which disappears as two successive layers come relatively to rest, takes the form of potential sheer energy, which reappears as kinetic in the return journey.

Displacement Diagram for Longitudinal Waves.—There is a most valuable graphic method of representing air-waves which readily enables us to realise the state of affairs in different parts of a wave.

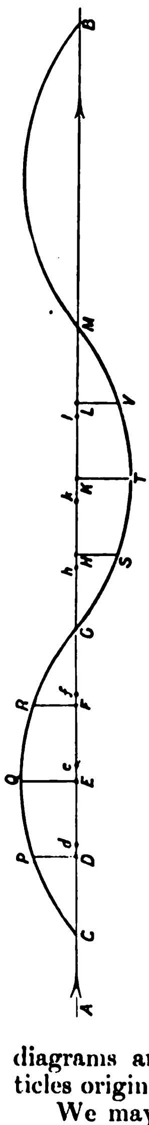

Fig. 8.—Displacement Diagram for Longitudinal Waves.

CDEFG . . . . original positions.
CdefG . . . . positions at a particular instant.
CPQRG . . . curve with height representing the displacement of each particle at that instant.

Let AB, Fig. 8, be the direction of propagation of the waves, and let C, D, E, F, G . . . be the original undisturbed position of certain particles in this line. At a particular instant, when the waves are passing, let these particles occupy the positions C, *d*, *e*, *f*, G, *h*, *k* . . . From each point originally occupied by a particle draw a perpendicular to AB in the plane of the paper, and proportional to the displacement of that particle, above AB if the displacement is forward, and below it if backward. Thus DP, EQ, FR are drawn upwards respectively proportional to D*d*, E*e*, F*f*, while HS, KT, and LV are drawn downwards respectively proportional to H*h*, K*k*, L*l*. The points CGM being in their original positions, the perpendiculars are there zero. If the construction is carried out for every point in AB, the ends of the perpendiculars will form a curve CPQRGSTVM, which fully represents the state of displacement in the waves on some chosen scale; in other words, it represents their **form**. We shall call a figure so drawn the **displacement diagram** for the waves. The reader will probably avoid confusion of thought if he carefully bears in mind from the first that the displacement diagram is merely a conventional representation of the waves, the representation of the displacement being at right angles to the actual displacement. Only in the case of transverse waves with the disturbances all in one plane—plane-polarised waves—can the diagram represent the actual displacement as it occurs. It is very instructive to draw several different displacement diagrams and then mark the actual displaced positions of particles originally equidistant along the line of propagation.

We may at once deduce from the displacement diagram the distribution of pressure throughout a wave. Thus, about the point E the pressure has its normal value, as the neighbouring particles are all displaced by the same amount forwards, the curve being parallel to AB. At K, again, the pressure is similarly normal. But between E and K there is an excess of pressure, since the particles from E to G are displaced, each one farther forward than the next; and those from G to K are displaced, each one less backward than the next. They therefore crowd on each other. From C to E and from K to M the particles are drawn away from each other, and therefore the pressure is less than the normal.

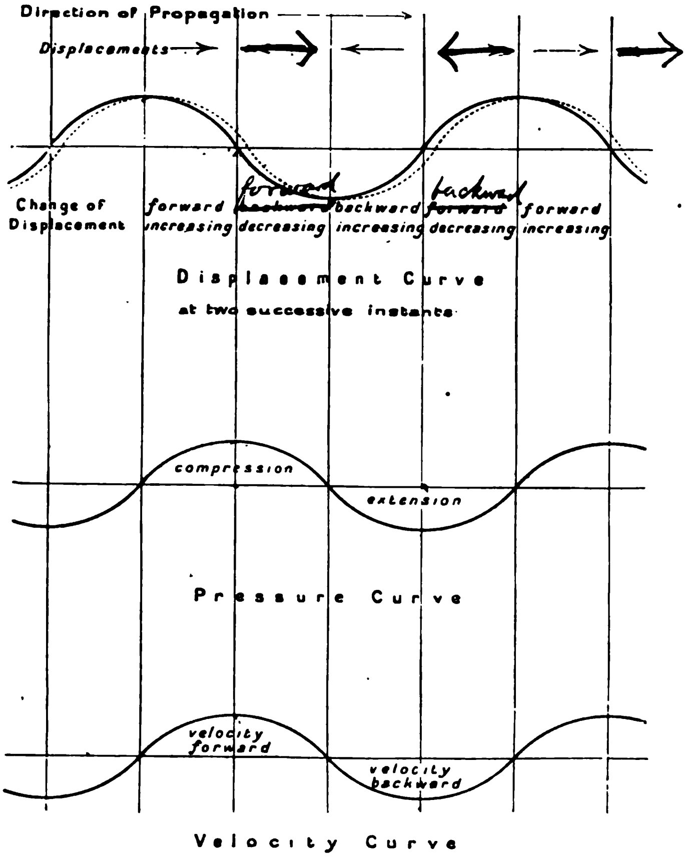

FIG. 9.—Displacement Diagram in two successive instants. The Pressure and Velocity Curves deduced from it.

All the phenomena of sound justify us in assuming that the waves of disturbance travel on, the same in form and without breaking down, so that a moment afterwards the whole displacement diagram is shifted forward a little, though otherwise unchanged. This at once enables us to determine the distribution of velocities, by noting whether a given ordinate of the curve will grow or lessen by the shifting, and whether it will be a change forwards or backwards. Fig. 9 represents a displacement diagram at two successive instants, and underneath are the pressure and velocity diagrams deduced from it. It appears at once that forward velocity accompanies compression, and backward velocity extension.

Wave-Length.—In a train of waves, the wave-length—usually denoted by λ—is the length occupied before the disturbance begins to repeat itself. In Fig. 8 it is the length CM. If U is the velocity with which the waves move forward, and n the number which pass a point in one second, then a length U of the displacement diagram contains n waves, and

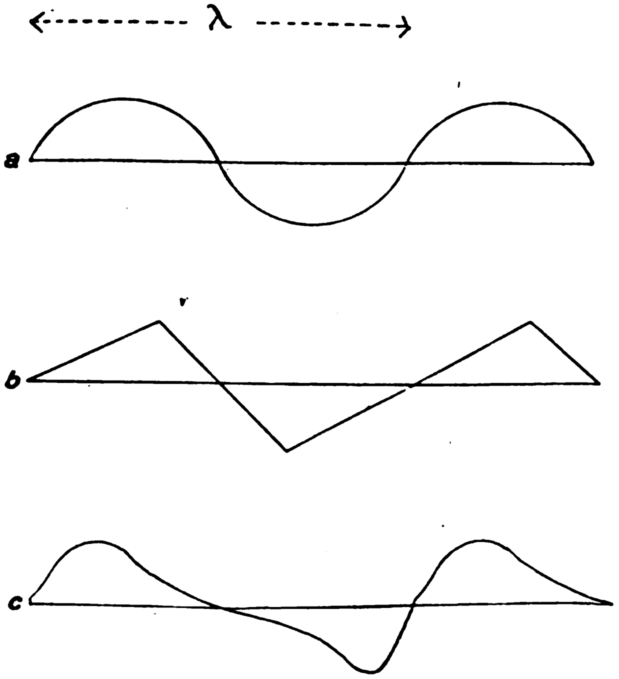

Fig. 10.—Equal Wave-Lengths and Amplitudes, but different forms. (*a*) tuning-fork; (*b*) violin; (*c*) open organ-pipe.

$$U = n\lambda.$$

If an observer is at the point, n is the frequency of the note he hears.

Phase.—The particular part of a wave at a point at a given instant is termed the phase of the disturbance at that point.

The phase evidently repeats itself after the passage of a wave length.

Comparing the displacement diagrams for two sets of waves, we see that they may differ from each other in three, and only three respects: in **amplitude**, **wave-length**, and **form**. We have already connected amplitude with loudness, and wave-length, as above, with pitch. On the assumption that the waves move on unchanged in form with constant velocity, we have only the third characteristic of form to connect with quality, and the conclusion that quality depends on form is, as we shall see hereafter, abundantly justified. In Fig. 10 there are three forms of wave, all of equal amplitude and length, but very different in form. These may be taken as representing, very nearly, the waves issuing respectively from a tuning-fork, a violin, and an organ-pipe when sounding the same note.

Noise.—Noise, as distinguished from musical sound, evidently corresponds to disturbance in which there is no regularity of alternation and no definite wave-length. Probably, too, when there is a mixture of a great number of waves of different lengths from different sources and without relation to each other, the ear, unable to pick out any one set of waves, hears only noise. Thus, when a number of people are talking together in a room, a bystander, listening to no one in particular, hears merely a noisy buzz. But many noises have some approach to pitch, and can at least be set down as high or low. Perhaps there may be some regular wave-motion accompanying the general irregular disturbance; thus, when an iron rod falls on the ground there is a noise, but accompanying the noise a note due to the vibration of the rod. In other cases it is possible that the sense of pitch may be due to a secondary disturbance arising in the auditory apparatus of the observer or in the connected air passages.

CHAPTER II.

THE VELOCITY OF SOUND IN AIR AND OTHER MEDIA — REFLECTION AND REFRACTION OF SOUND.

CONTENTS.—Velocity of Waves in Air—Determinations of the Velocity of Sound in Air and other gases by Direct Experiment—Velocity of Sound in Water and in Isotropic Solids—Reflection of Sound—Refraction of Sound.

THE identification of sound in air with waves of longitudinal disturbance is most satisfactorily established by the almost perfect agreement between the observed velocity of sound and the calculated velocity of waves, the calculation being based on the known mechanical properties of air.

In this chapter we shall first determine the velocity on certain suppositions which considerably reduce the mathematical difficulties of the subject, and we shall then give an account of various experiments which have been made to determine the velocity of sound.

Velocity of Longitudinal Waves in Air.—We shall assume: (1) that the displacement is small compared with the length of a wave—in other words, that the displacement diagram, if drawn on the scale 1 : 1, is very nearly flat; and (2) that the state as to displacement is the same at the same instant in a plane of great extent, drawn perpendicular to the line of propagation. Let the displacement diagram H P Q K represent the longitudinal disturbance existing at a given instant along the line AB, Fig. 11, the waves travelling from A towards B. We shall suppose that an external force is applied to every particle of air, such that, in conjunction with the internal force, due to pressure variation, it just suffices to propagate the displacement unchanged in form with velocity, U. Let the external force on an element of cross-section 1 per length 1 be X. The internal force is the difference between the two end-pressures at M and N. If the undisturbed pressure is P, we may denote these by $P+p_M$ and $P+p_N$ respectively, and the internal force is $p_M - p_N$.

The bulk-elasticity, E, of the air is the ratio of a small increment of pressure, to the corresponding decrement of volume per unit volume, and is constant for small changes, such as are implied in assuming that the curve of displacement is nearly flat. This implication will be evident from the expression for the volume change given below.

We therefore have

$p_M = E \times$ change of volume per unit volume at M from the normal condition.

If the change of volume were everywhere the same as at M, each layer would be crowded forward by the same amount on the next, and the displacement curve would be represented by PT, the tangent at P. Now the original volume of the air between M and L was $ML \times 1^2$. But M is moved forward MP, and L is moved forward LT, or M gains on L by PS. Hence the change of volume in ML is $PS \times 1^2$, and the change per volume 1 is $\frac{PS}{ML}$ or $\frac{PS}{ST}$.

We may therefore put—

$$p_M = E \, . \, \frac{PS}{ST} \text{ and } p_N = E \, . \, \frac{TV}{VQ} \quad . \quad . \quad . \quad (1.)$$

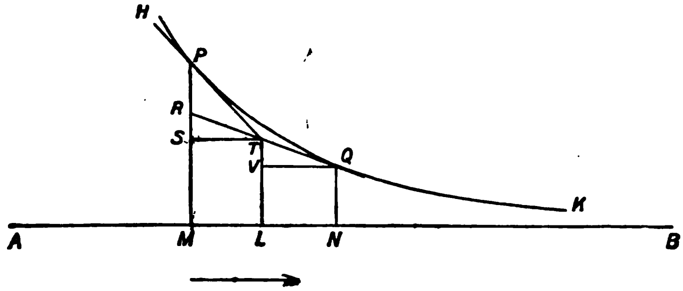

FIG. 11.—Displacement Curve for a longitudinal disturbance, all the ordinates being small. TP, TQ, tangents at neighbouring points P, Q.

Relation between Particle Velocity and Pressure Excess.—There is an important relation between the velocity of a particle and the excess of pressure to which it is subjected, which we shall use in Chapter VII. as well as here. Denoting by u_N the velocity of the particle of air at N, the displacement due to this velocity would evidently change from QN to TL or by TV, while the disturbance passed over LN, *i.e.*, in the time LN/U.

Then

$$TV = u_N \, . \, \frac{LN}{U}$$

or

$$\left.\begin{aligned} \frac{u_N}{U} &= \frac{TV}{LN} = \frac{TV}{VQ} = \frac{p_N}{E} \\ \text{Similarly} \qquad \frac{u_M}{U} &= \frac{p_M}{E} \end{aligned}\right\} \quad . \quad . \quad . \quad (2.)$$

Acceleration and Force.—The velocity of the particle at N changes from u_N to u_M while the disturbance travels over MN, *i.e.*, in time MN/U. Hence

$$\text{acceleration} = \frac{u_M - u_N}{\frac{\text{MN}}{\text{U}}} = \text{U} \, . \, \frac{u_M - u_N}{\text{MN}}$$

which from (2)

$$= \frac{\text{U}^2}{\text{E}} \; \frac{p_M - p_N}{\text{MN}} \quad . \quad . \quad . \quad . \quad . \quad (3.)$$

The force on the element of length MN and cross section 1^2 is

$$p_M - p_N + \text{X} \, . \, \text{MN}$$

Dividing this by the mass $\rho \, . \, \text{MN}$ we may equate to the acceleration (3) and we have

$$\frac{p_M - p_N}{\rho \, . \, \text{MN}} + \frac{\text{X}}{\rho} = \frac{\text{U}^2}{\text{E}} \, . \, \frac{p_M - p_N}{\text{MN}}$$

and

$$\text{U}^2 = \frac{\text{E}}{\rho}\left(1 + \text{X} \, . \, \frac{\text{MN}}{p_M - p_N}\right) \quad . \quad . \quad . \quad . \quad . \quad (4.)$$

If $\text{X} = 0$ at every point, that is, if there is no external force, and if the internal pressure changes are proportional to the volume changes, so that E is constant,

$$\text{U}^2 = \frac{\text{E}}{\rho}$$

or

$$\text{U} = \sqrt{\frac{\text{E}}{\rho}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5.)$$

If, then, we once start longitudinal waves of the kind supposed, with the velocity given by (5) they will be propagated unchanged with that velocity.[1]

Newton's Value of U.—The result $\text{U} = \sqrt{\text{E}/\rho}$ was first obtained by Newton.[2] In working it out, he assumed—

increase of pressure/original pressure =
decrease of volume/original volume.

This is equivalent to Boyle's law. For, according to that law, if P, V are the original pressure and volume, and $\text{P} + p$, $\text{V} - v$ their new values:

$$\begin{aligned} \text{PV} &= (\text{P} + p)(\text{V} - v) \\ &= \text{PV} + p\text{V} - \text{P}v - pv. \end{aligned}$$

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[1] If the pressure changes are too considerable to justify the assumption that they are proportional to the volume changes, we may regard the variation from proportionality as an external force represented by X. Thus, in a wave of very considerable displacement and pressure excess, $\text{X}/(p_M - p_N)$ can be shown to be positive, and U is greater than the value in (5). This agrees with certain experimental results given below.

[2] *Principia*, Bk. ii. sect. 8, Props. 47 and 49.

Neglecting pv as the product of two exceedingly small quantities, we have

$$Pv = pV,$$

whence

$$E = \frac{pV}{v} = P.$$

Using this value, Newton, with the data at his command, found

$$U = 979 \text{ ft./sec.}$$

If we take the atmospheric pressure as 1,016,000 dynes/sq. cm., and the density of the air as ·001293 at that pressure, we obtain

$$U = 28{,}000 \text{ cm./sec., say } 920 \text{ ft./sec.}$$

the difference between 979 and 920 being due to Newton's imperfect data.

Now direct experiment gives (nearly) $U = 33{,}000$ cm./sec., say 1080 ft./sec., a value greater by about $\frac{1}{6}$. The discrepancy, according to Newton, was about $\frac{1}{8}$. Newton sought to explain the discrepancy by supposing that the molecules occupied about $\frac{1}{8}$ of the linear distance traversed, and that the sound passed instantly through these, only taking time to go through the interspaces. He also supposed that the vapour present in the air, which he took as $\frac{1}{11}$ of the whole, had no part in the motion, and thus he arrived at a value 1142 ft./sec. There was, however, no justification whatever for either of these suppositions.

Laplace's Correction.—It was not till 1816 that the source of error was detected, when Laplace pointed out that the elasticity was not to be found from Boyle's law. In sound-waves the compressions and rarefactions take place in such quick succession that there is not time to share with the surroundings the heat developed by the compression or the cold developed by the rarefaction. Now Boyle's law would only hold if the temperature remained uniform, and this would require the changes of pressure to be made so slowly that changes of temperature would be prevented by conduction and radiation to or from the surrounding air. The result of rise of temperature in compression is that a greater increase must be given to the pressure for a given volume decrease than when the temperature is constant. Similarly, a fall of temperature in rarefaction requires a larger decrease of pressure for a given extension, and so the elasticity is greater in both cases. Laplace supposed that the changes occur in such rapid succession that there is no sensible passage of heat into or out of the waves. Perhaps a better statement of the condition is that the temperature slope is so small that no appreciable fraction of the temperature difference is removed by conduction or radiation while a single wave is passing. The changes are then adiabatic, and the relation between pressure and volume for a gas is given by

$$PV^\gamma = \text{constant},$$

where γ is the ratio Specific Heat at constant pressure / Specific Heat at constant volume. From this law it can easily be shown that

$$\frac{p}{P} = \gamma \frac{v}{V}$$

if p and v are very small; whence the adiabatic elasticity

$$E = \frac{pV}{v} = \gamma P,$$

a result which is in accordance with the general rule

$$\text{Adiabatic elasticity} = \gamma \,.\, \text{Isothermal elasticity.}$$

Various experimental determinations of γ have been made,[1] and the value for air is certainly not far from 1·40. Using this value in

$$U = \sqrt{\gamma P/\rho},$$

we get $U = 33{,}170$ cm./sec. approximately, which agrees very closely with the best experimental determinations described below.

This agreement appears to justify Laplace's supposition that the changes of volume are adiabatic, and this supposition has been entirely verified by Stokes, who has shown that the elasticity must either be the isothermal one used by Newton or the adiabatic one used by Laplace. With any intermediate condition there would be dissipation of energy, such that the waves would die away with extreme rapidity. This dissipation may be illustrated by considering the case of a cylinder containing air and closed by a piston working quite smoothly in the cylinder. If the piston is projected in one direction, it will oscillate to and fro, alternately compressing and rarefying the air. If the time of oscillation is so great that no temperature changes occur, there will always be the same pressure on the piston in the same position, and the oscillation will continue without dissipation of its energy. If the time of oscillation is so small that the changes are adiabatic, the pressures will again be definite in definite positions, though not the same as in the previous case, and there will again be no dissipation. But if the changes are neither adiabatic nor isothermal, during a given compression some of the heat will leak away, and therefore, during the following extension, the temperature of the air at a given volume will be lower than it was for the same volume during the compression; the pressure against the piston will be less, and the whole of the kinetic energy given up by the piston in compression will not be restored in extension. In fact, the energy has been dissipated by conduction from a higher to a lower temperature. It may be pointed out that there is also conduction of heat in the isothermal change, but as this is between bodies at practically the same temperature, there is no appreciable dissipation of energy.

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[1] For an account of these, see *Heat.*

The Change of Velocity U with Change of Temperature.—If we assume the equation which is approximately true for gases—

$$P = \kappa\rho(1 + at),$$

where κ is a constant for the same gas and $a = .00367$, we get

$$U = \sqrt{\gamma P/\rho} = \sqrt{\gamma\kappa(1 + at)}.$$

If $t = 0°$,

$$U = U_0 = \sqrt{\gamma\kappa},$$

whence, in general,

$$U = U_0\sqrt{1 + at}.$$

If t is small, this becomes

$$= U_0\left(1 + \frac{at}{2}\right)$$

$$= U_0(1 + .00184t).$$

For dry air this is, in centimetres per second,

$$33{,}200 + 61t.$$

It is evident from this result that the velocity depends on the temperature and not on the pressure of the air, so long as the constitution is the same. Of course, if ρ varies for the same values of P and t, then U will be affected. If, for instance, the humidity increases, ρ diminishes and U increases.

Application of the Formula to Liquids.—Since the only assumption made in the investigation on which the formula is based is that the waves are of the longitudinal type, we may apply the result to liquids as well as gases, and we obtain

$$U = \sqrt{E_\phi/\rho},$$

where E_ϕ is the adiabatic elasticity.

Direct experiment gives us the isothermal elasticity E_θ and here also $E_\phi/E_\theta = \kappa_p/\kappa_v$ where κ_p and κ_v are the specific heats respectively at constant pressure and constant volume. For liquids, however, the relation between κ_p and κ_v is not so simple as for gases, but we have, in mechanical units,

$$\kappa_p - \kappa_v = a^2 V \theta E_\theta,$$

where a is the co-efficient of volume increase, V is the volume of unit mass, and θ is the absolute temperature.

In the case of water, we know all the quantities a, V and E_θ. Thus, according to Grassi,[1] at 4°, $E_\theta = 2{\cdot}03 \times 10^{10}$; and at 15°, $E_\theta = 2{\cdot}23 \times 10^{10}$. But at 4°, $a = 0 \therefore \kappa_p = \kappa_v$ and

$$U = \sqrt{E_\theta/\rho} = 142{,}500 \text{ cm./sec.};$$

At 15°,

$$a = {\cdot}00016,$$
$$V = 1,$$
$$\theta = 288,$$
$$\kappa_p = 4{\cdot}16 \times 10^7;$$

whence

$$\frac{\kappa_p - \kappa_v}{\kappa_p} = {\cdot}00395$$

and

$$\frac{\kappa_p}{\kappa_v} = 1{\cdot}004 \text{ (approx.)},$$

$$\therefore \qquad U = 149{,}600 \text{ cm./sec.}$$

The most trustworthy experiments yet made (see below, p. 30) give us the velocity of sound in water at 8·1° C.—

143,500 cm./sec.

This is as close an agreement as could be expected.

DETERMINATIONS OF THE VELOCITY OF SOUND BY DIRECT EXPERIMENT.

We shall now give a brief account of some of the more interesting and important experiments which have been made to determine the velocity of sound in air.

Corrections to Dry Air at 0° C.—As the velocity is not fixed, but is dependent on qualities which are continually varying, it has been usual, in all but the earlier experiments, to observe the state of the air as to humidity and temperature, and hence from the observed velocity to calculate the velocity in dry air at 0°, assuming

$$U = \sqrt{\gamma P/\rho}.$$

Thus if ρ_0 is the density of dry air at P and 0°, while ρ is the density of the air at the time of the experiment

$$U_0 = \sqrt{\gamma P/\rho_0},$$

whence

$$U_0/U = \sqrt{\rho/\rho_0}$$

and

$$U_0 = U\sqrt{\rho/\rho_0}.$$

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[1] Violle, *Cours de Physique*, i. p. 525.

Effect of Wind—Derham's Experiments.—The velocity calculated is that relative to a starting-point in the air itself, and is only relative to a point fixed on the surface of the earth when the air is at rest. When the air is moving, the waves are carried with it, and the velocity relative to the surface of the earth—the velocity observed—is the resultant of the velocity relative to the air and the velocity of the wind.

The effect of wind was first established by a long series of experiments by Derham at the beginning of the last century on the time taken by the report of a cannon fired on Blackheath to reach the tower of Upminster Church in Essex, a distance of 12½ miles across the Thames. The time taken by the flash to travel over this length being negligible, the interval between flash and report was that taken by the sound. The interval varied between 55½ and 63 seconds, according as the wind and sound travelled in the same or in opposite directions. Derham obtained as the velocity of sound 1142 ft./sec.[1] Believing that temperature and humidity were without effect, he did not record these.

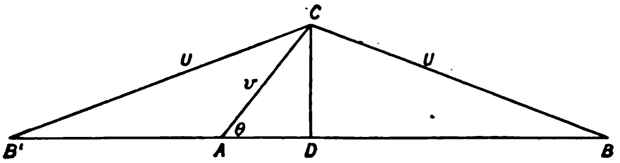

FIG. 12.—Velocity of Sound in Wind. AC, velocity of wind. CB = CB' = U, AB, AB', velocities of sound in directions AB, AB'.

Method of Correction by "Reciprocal" Observations.—In most of the later experiments the time of passage of the sound between two points has been taken in both directions, and, by taking the mean, the effect of wind has been to a great extent eliminated. For let B'AB, Fig. 12, represent the direction of the line joining the two points, and let $AC = v$, making θ with AB, represent the velocity of the wind. Draw $CB = CB' = U$, the velocity of sound in still air. Then AB, AB' will represent the velocities of the sound in the two opposite directions respectively.

If we denote them by V_1 and V_2, drawing CD perpendicular to BB':

$$V_1 = BD + AD$$
$$V_2 = BD - AD.$$

But

$$BD = \sqrt{BC^2 - CD^2} = \sqrt{U^2 - v^2 \sin^2 \theta}$$

and

$$AD = v \cos \theta.$$

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[1] *Phil. Trans.*, 1708; Hutton, v. p. 380.

Expanding and neglecting powers of v/U above the second

$$V_1 = U\left(1 + \frac{v\cos\theta}{U} - \frac{v^2\sin^2\theta}{2U^2}\right)$$

$$V_2 = U\left(1 - \frac{v\cos\theta}{U} - \frac{v^2\sin^2\theta}{2U^2}\right).$$

If T_1 and T_2 are the times of passage over the distance L between the two points, and T their mean :

$$T_1 = \frac{L}{V_1} \qquad T_2 = \frac{L}{V_2}$$

$$\therefore \quad T = \frac{T_1 + T_2}{2} = \frac{L}{2}\left(\frac{1}{V_1} + \frac{1}{V_2}\right)$$

$$= \frac{L}{2U}\left\{\left(1 + \frac{v\cos\theta}{U} - \frac{v^2\sin^2\theta}{2U^2}\right)^{-1} + \left(1 - \frac{v\cos\theta}{U} - \frac{v^2\sin^2\theta}{2U^2}\right)^{-1}\right\}$$

$$= \frac{L}{U}\left\{1 + \frac{v^2}{2U^2}(1 + \cos^2\theta)\right\},$$

or

$$U = \frac{L}{T}\left\{1 + \frac{v^2}{2U^2}(1 + \cos^2\theta)\right\}.$$

Probably $\frac{v}{U}$ is never more than $\frac{1}{10}$ in the weather selected for experiments, so that $\frac{v^2}{U^2}$ is negligible, and therefore $U = \frac{L}{T}$.

This, of course, assumes that the wind is constant during the two opposite passages. No doubt there are variations in reality, and their effect can only be eliminated by repetition.

Experiments of 1738.—In 1738 a commission of the French Academy of Sciences made experiments between Montmartre and Montlhéry, a distance of 17 or 18 miles, observers being stationed at the Observatory of Paris and the Château de Lay, which were in the line joining the extreme points. Cannons were fired at half-hour intervals alternately at the two ends of the line, and the intervals between flash and report were noted by the observers at the various stations. Though the sound was not always transmitted all the way, data were obtained which gave as the actual velocity in modern measure:

$$U = 337 \text{ met./sec.}$$

The temperature was about 6° C., whence

$$U_0 = 332 \text{ m./sec.}$$

The Experiments of 1822.—After Laplace had pointed out the source of error in Newton's calculations, a Commission was appointed by the Bureau des Longitudes which experimented again on the outskirts of Paris, between Montlhéry and Villejuif, a distance of about 11 miles. Reciprocal cannon-firing was used, but at intervals of only five minutes, and chronometers replaced the pendulum clocks of the first experiment. At 15·9° C. the result was $U = 340{\cdot}9$ m./sec., whence

$$U_0 = 331 \text{ m./sec.}$$

The Experiments of Moll and Van Beck.—In 1823 these observers made a determination of the velocity over a distance of 17,000 metres between Levenboompjes and Kooljesberg, near Utrecht, reducing the interval between firing at the two ends to one second. They found

$$U_0 = 332{\cdot}8 \text{ m./sec.}$$

Experiments at Different Levels.—In 1823 Stampfer and Myrbach made determinations between two stations in the Tyrol differing in level by 1364 metres. In 1844 Bravais and Martin experimented between two stations on the Faulhorn differing in level by 2079 metres. Both pairs of observers found:

$$U_0 = 332{\cdot}4 \text{ m./sec.}$$

These experiments are interesting in that they fully confirm the result of the calculation that the velocity is independent of pressure.

Experiments at Very Low Temperatures.—Determinations have been made in Arctic expeditions at temperatures below the freezing-point. Thus in Parry's third voyage the velocity was determined at temperatures ranging from −38·5° F. to −7° F., and again at 33·5° and 35°.

From the actual results (see Airy's *Sound*) the velocity at 0° F. would be about 1050 ft./sec., and the temperature correction about 1 ft./sec. per degree F. This agrees fairly with calculation.

A large number of observations were made by Greely[1] at temperatures between −10° and −45° C., giving in m./sec.

$$U = 333 + {\cdot}6t,$$

which agrees very closely with the formula of p. 21.

Stone's Experiments.—In 1871 a determination was made by Mr. Stone at Cape Town.[2] Two observers were stationed, one at a distance of 641 feet from the one-o'clock time-gun at Port Elizabeth, the other at the Observatory, 15,449 feet away. On hearing the report, these observers gave electric signals recorded on the chronograph at the Observatory. As there was no reciprocal firing, the wind velocity was measured and allowed for.

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[1] *Phil. Mag.*, 1890, xxx. p. 507.

[2] *Ibid.*, 1872, xliii. p. 153.

The time taken by the electric signals being negligible, the time between the records on the chronograph should have given the time taken by the sound to travel the distance between the observers.

But the chronograph records were now corrected on account of "personal equation." Every observer takes a greater or less time to record any event which he observes, and the interval between the actual time at which the event, as it were, reaches him, and the time at which he records it, is termed his **personal equation**. If, in the experiments at Cape Town, the observers had had exactly equal personal equations, the recorded would have been equal to the true time; or if they had had unequal but constant personal equations, the effect might have been eliminated by interchanging the observers. But in all probability the personal equation varies with the same observer, depending, among other conditions, on the intensity of the sound, being greater for faint than for loud sounds. In order, then, to eliminate the effect, a subsidiary experiment was made, in which a gun was fired 1483 feet from the Observatory, and 162 feet from the first observer, the loudness for each observer being estimated as equal to that of the time-gun in the main experiment. The time taken by the sound to travel over so short a distance could be calculated without serious error from that over the ten times greater distance without considering the personal equation effect, which would be reduced to $\frac{1}{10}$th.

The recorded interval was greater than that calculated by ·09 sec., and therefore ·09 sec. was subtracted from the intervals recorded from the longer distance, as the excess of the personal equation of one observer over that of the other. It is interesting to note that when the observers were interchanged in the subsidiary experiment the error was ·02 only, but still in the same direction, tending to show that the farther observer had always the larger equation, but that one had a greater equation than the other at this distance. The final value obtained by Stone was:

$$U_0 = 1090{\cdot}6 \text{ ft./sec.}$$

or

$$332{\cdot}4 \text{ m./sec.}$$

Regnault's Experiments.—Before Stone had shown the practical importance of the personal equation, Regnault had devised and carried out a method in which the sound-wave was made to give its own record by means of electric signals.[1] He determined the velocity not only in the open air, but also in pipes of various diameters.

The open-air experiments were made in 1864 at the Polygone de Satory near Versailles over two distances, respectively 2445 and 1280 metres. The source of sound was a gun, and reciprocal firing was adopted. To record the instant of firing, a wire forming part of an electric circuit was stretched across the muzzle of the gun. This was broken by the explosion, and the interruption in the circuit was recorded on the revolving drum of a chronograph.

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[1] *Phil. Mag.*, 1868, xxxv. p. 161.

At the distant station the sound-wave was received by a wide cone fixed on to a cylinder, closed at the farther end by a thin india-rubber membrane. When the wave impinged on the membrane, it was driven forward and an electric circuit was broken, the interruption being recorded on the chronograph, on which the firing had already been marked.

At first it might appear that personal equation was thus eliminated, but in fact the membrane has as much a personal equation as an animate observer. For it will take some time to acquire energy from the wave, and inevitably there will be delay in responding to the sound. Regnault sought to measure and allow for this delay by special experiments.

The values for the velocity given by the mean of a great number of experiments were—

Over 1280 metres, $U_0 = 331{\cdot}37$ m./sec.
„ 2445 „ $U_0 = 330{\cdot}7$ m./sec.

This falling off in velocity with increase of distance is in agreement with various other experiments, all tending to show that the velocity is greater with greater intensity, and that it only tends to a lower limit as the intensity diminishes. If we suppose that the sound travels over the 1165 metres between the 1280 and 2445 metres from the source, with the limiting velocity, we find

$$U_0 = \frac{1165}{\dfrac{2445}{330{\cdot}7} - \dfrac{1280}{331{\cdot}37}} = 329{\cdot}9 \text{ m./sec.}$$

In the years 1862–66 Regnault made a very extensive series of researches by the same method on the propagation of sound in tubes, using water and gas pipes and tubes set up in his laboratory of various lengths up to 4900 metres, and various diameters from 1·08 decimetres to 1·1 metres.[1] In the longer tubes, intermediate receiving stations were sometimes used, and in all cases the wave could be followed through several passages to and fro in the pipe after reflection at the ends. The receiving drum now made an electric circuit, instead of breaking it as in the open-air experiments. The drum had, unfortunately, a personal equation, increasing as the sound grew more faint; and though this was allowed for, it is to be feared that its effect was not wholly eliminated, and that Regnault's results are slightly under the true value.

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[1] Violle, *Cours de Physique*, ii. A. p. 64.

Various sources of sound were used, including a pistol with varying charges, an explosion of a mixture of oxygen and hydrogen, and musical instruments. In all cases temperature and humidity were observed and allowed for. The following results were obtained:—

1. The velocity decreases with decreasing intensity, tending towards a lower limit for very feeble sounds. Thus the velocity of the wave from a charge of 1 gramme of powder in the widest tube sank from 334·16 to 330·52 m./sec. as it travelled to and fro.

2. The velocity increases with the diameter, tending to a limit with very wide tubes. Thus, in the tube ·108 metre in diameter, it is 324·25 m./sec., and in that 1·1 metres in diameter it is 330·3 m./sec. This agrees with a formula found by v. Helmholtz, and afterwards modified by Kirchoff,[1] according to which the velocity should be

$$U = U_0\left(1 - \frac{c}{D\sqrt{N}}\right),$$

where U_0 is the open-air velocity,
 D . . . diameter,
 N . . . frequency of the note sounded,
and c a constant, having different values according to the two investigators.

3. The limiting velocity is the same for all sources.

4. The velocity is independent of the pressure. Thus in the laboratory experiments the same value was obtained with pressures varying from 247 mm. to 1267 mm.

Regnault gave, as the final result of his experiments, the value for a feeble sound in an infinitely wide tube

$$U_0 = 330{\cdot}6 \text{ m./sec.}$$

Experiments were also made on the velocity of sound in different gases by means of the tube set up in the laboratory. According to our calculation, the ratio of the velocities U, U′ in two gases of densities ρ, ρ' at the same temperature and pressure should be

$$U'/U = \sqrt{\rho/\rho'}.$$

Regnault found, taking U as the velocity in air

	U'/U	$\sqrt{\rho/\rho'}$
Hydrogen	3·801	3·682
Carbon dioxide	0·7848 0·8009	0·8087
Nitrogen	0·8007	0·8100
Ammonia	1·2279	1·3025

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[1] Rayleigh's *Theory of Sound*, vol. ii. §§ 349, 350.

The first two numbers were obtained from a tube more than 500 metres long, and the gas was perhaps not maintained pure in such a length. In the others, the length was 70 metres.

Other Experiments showing Variation of Velocity with Intensity.—Violle and Vautier[1] working with a tube ·7 metre in diameter, and using Regnault's apparatus with certain improvements, obtained results agreeing with the formula

$$\text{Vel.} = U_0(1 - C\sqrt{P}),$$

where P is the mean variation from the normal pressure in a wave, and C a constant. They found the same velocity for different notes, perhaps because the tube was wide. Jacques, experimenting at Watertown, Mass.,[2] has shown that when a cannon is the source of sound, the velocity varies in different directions round it. He used membranes as receivers, recording on a chronograph, and found that "immediately in the rear of the cannon the velocity was less than at a distance, but that going farther and farther from the cannon *the velocity of sound rose to a maximum considerably above the ordinary velocity, and then fell gradually to about the velocity usually received.*" Thus, with a charge of 1½ lbs. of powder, the velocity rose from 1076 ft./sec. at 10 to 30 feet in the rear to a maximum of 1267 ft./sec. at 70 to 90 feet in the rear and then fell off.

Perhaps, owing to the fact that the intensity, and therefore the velocity, in front and at the sides is much greater than at the back, the wave-surface will have at first a kind of dimple at the back. This may be of such form that it fills in, as it were, from the sides, and ultimately disappears. If so, on drawing successive positions of the surface, it will be seen that the results observed by Jacques are probably explained.

The Krakatoa Eruption.—A very remarkable example of sound-waves on a gigantic scale was given by the Krakatoa eruption on August 26–27, 1883.[3] This was accompanied by a series of explosions, culminating in a tremendous outburst on the morning of August 27, when a large part of a mountain was blown up into the air. The sounds were heard over an area estimated at one-thirteenth of the earth's surface, and at more than 2000 miles away they were like the firing of heavy guns. The wave due to the chief explosion as it travelled round the globe affected all the barometers and left a very noticeable trace in the self-recording instruments. By a careful examination of these records, it was found that the wave travelled at about 700 miles per hour (the velocity of sound at 0° F. is 723 miles per hour); that it culminated at the side of the earth opposite to

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[1] *Phil. Mag.*, 1888, xxvi. p. 77.

[2] *Phil. Mag.*, 1879, vii. p. 219.

[3] "The Eruption of Krakatoa," Royal Society Report; *Nature*, vol. xxxviii. 1888, p. 566.

Krakatoa in 18 hours; that it then spread out again, culminated again, and so on, the trace not being entirely lost for 127 hours.

Velocity of Sound in Water.—In 1826, MM. Colladon and Sturm determined the velocity of sound in Lake Geneva. The source was a bell struck by a lever about a metre below the surface. By the same blow some gunpowder was ignited, giving a flash which could be seen at the distant station, as the experiments were made at night. The sound was received by a kind of ear-trumpet—a tin tube about 3 metres long. The lower end, 2 decimetres in diameter, was closed and plunged under water, and the upper end was applied to the ear of the observer. The stroke of the bell could be heard even across the lake, a distance of 14,000 metres or 9 miles. The interval between flash and sound was registered on a quarter-second stop-watch, and the mean velocity found from a number of determinations was 1435 per second, the mean temperature of the water being estimated at 8·1° C.

Velocity of Waves from Explosions in Water.—Messrs. Threlfall and Adair have shown[1] that the velocity of a wave sent out by an explosion in sea-water increases very greatly with the intensity of the explosion. Various charges of gun-cotton were fired under water in the harbour of Port Jackson, Australia, and the time taken by the wave to travel 150 or 180 metres was measured. The charge was fired by an electric current, which gave a signal recorded on a pendulum chronograph, and the receiver was an india-rubber diaphragm several feet under water, so arranged that on disturbance it completed an electric circuit recording on the chronograph. The calculated velocity of sound was about 1500 metres per second; the observed velocity of the explosion-wave rose from 1732 met./sec. with 9 oz. of gun-cotton to 2013 met./sec. with 64 oz.

Velocity of Sound in Isotropic Solids.—Several kinds of waves are possible in solid bodies. In solids extended in every direction we may have waves of longitudinal disturbance propagated with velocity—

$$U = \sqrt{(\kappa + \tfrac{4}{3}n)/\rho},$$

where κ is the bulk modulus of elasticity and n the rigidity; and we may have waves of transverse disturbance propagated with velocity—

$$U = \sqrt{n/\rho}.$$

The former would probably be termed sound-waves, if the subject were experimentally studied; but at present we may say that it has a theoretical interest only.

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[1] *Proc. Royal Society*, 1889, xlvi. p. 496.

In cords and thin bars we may have waves of longitudinal disturbance accompanied by lateral shrinkage or expansion, propagated with velocity—

$$U = \sqrt{Y/\rho}$$

where Y is Young's modulus, and is

$$= 9n\kappa/(3\kappa + n).$$

We may also have transverse waves, depending on the elasticity of the material only as the thickness becomes considerable, and torsional or twist waves. All these have been studied experimentally, and we shall enter into more detail later.

Indirect Measures of the Velocity of Sound by Rods and Pipes.—The velocity in tubes containing gases, and the velocity in solid rods and wires, may be determined indirectly by the notes emitted by given lengths. We shall give some account of the methods employed when we discuss the vibrations in pipes and rods (Chapters VII. and VIII.).

REFLECTION OF SOUND.

Examples of the reflection of sound in the form of echo are well known to every one, and they can always readily be explained by the presence of some reflecting surface, such as a hillside, a rock, or a wood. Certain echoes present peculiarities, which are doubtless to be referred to the nature and position of the reflecting surface. Sometimes, for instance, the quality of the reflected sound differs from that incident on the surface, a shout in a particular note coming back apparently an octave higher. The octave was present in the shout, but was masked by the predominant fundamental note. But probably the irregularities of the surface were comparable with the wave-length of the lower note and destroyed it or diffused it, while they were large compared with the wave-length of the higher note, and so it could be reflected from each of them.

In many rooms a particular note is echoed to and fro for a very appreciable time, more especially if the source is at a particular point. This is probably due to some relation between the dimensions of the room and the wave-length of the note, which is, perhaps, one of the upper partials of the room (Chap. VII. p. 120).

Frequently, a sharp sound, such as the clapping of the hands or of two boards together, is reflected in a room or a corridor with smooth, parallel walls as a more or less musical sound. Here the successive reflections reach the ear at definite intervals sufficiently near together to give a musical note.

Musical Echo from Palisading.—A similar effect is often observed when one is walking near palisading, each footstep of the observer being followed by a musical ring. The effect is only noted after some sudden sound, and may often be heard very distinctly on clapping the hands or on knocking two stones together. Let the vertical lines 1–8, fig. 13 *a*, represent the bars of the palisading. Suppose that an observer on the right has sent a very short wave, conventionally represented by a narrow arch, from right to left. As this passes each bar, a wave is sent out from the bar backwards. If at the instant chosen for the figure, the outgoing wave has just passed the last bar on the left, No. 8 (the outgoing wave is not shown), the reflected waves will be in the positions r_1, r_2, . . . r_8, the suffix denoting the bar from which reflection has occurred. These will form an equidistant series with well-marked periodicity, and the observer will hear a musical note. In the line of the palisades the distance between the successive waves is twice the distance between the bars, for while the incident wave moves forward, say from bar 7 to bar 8, the wave reflected from bar 7 moves back to bar 6, and the distance between r_7 and r_8 is two bars. Hence the wave-length is $2l$, where l is the distance between the bars, and if n is the frequency of the note heard, $2nl = U$, where U is the velocity of sound. It is worthy of special notice that the frequency of the note heard depends on the positions of the source and observer. Thus, with a source in the line of the palisading and an observer at right angles to that line, the note will be one octave higher.

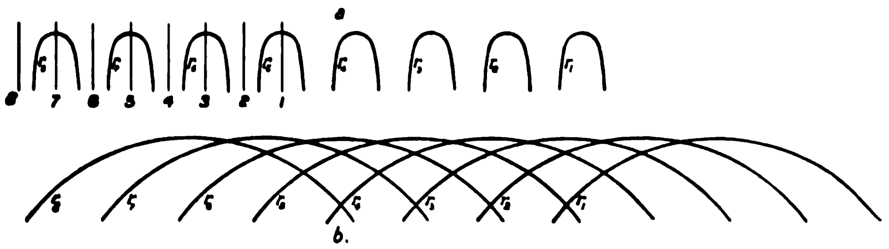

Fig. 13.—Reflection of a Single Wave from Palisading. *a*. short wave, forming a reflected series giving a musical note; *b*. long wave, forming a reflected series overlapping and without marked periodicity. 1-8 bars reflecting wave which has passed to the left of 8. $r_1 \ldots r_8$ series of reflected waves. Bars omitted in *b*.

If the incident wave is long, such as may be represented by a flat, wide arch, the reflections will be as in Fig. 13 *b*. The reflected waves will then overlap, and the periodicity will not be sufficiently defined to give an audible musical note.

Whispering Galleries.—Another very curious instance of the reflection of sound is given by whispering galleries, such as the well-known circular gallery in the dome of St. Paul's, where a whisper is transmitted round the surface of the gallery, so as to be perfectly audible all round near the wall. Lord Rayleigh explains this as chiefly due to a continued reflection of the sound at the surface.* Let S, Fig. 14, be the source of sound. Borrowing a useful term from optics, a "ray" of sound sent out along SP will be reflected so as to travel round in a series of equal chords, all touching the circle with radius ON, where N is the foot of the perpendicular from O on PSP′. If QSQ′ be the other tangent from S to this circle, a ray of sound sent out along SQ will also travel round in a series of equal chords touching the same circle. Intermediate rays will form shorter chords, and therefore travel round wider circles. Hence all the sound sent out between SP and SQ travels round outside the circle with radius ON; the lateral spreading is largely prevented, and the intensity remains sufficient to render the sound audible all round.

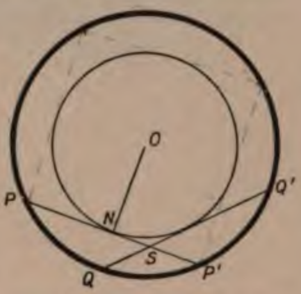

FIG. 14.—The Whispering Gallery. S, source; SP, SQ, two equal chords. All sound sent out between SP and SQ travels round outside the circle touching them.

Refraction of Sound.

Wind-Refraction.—It is a common observation that sound carries better with the wind than against it. Bells, trains, and other notable sources are often heard when the wind is from their direction, though they may be quite inaudible at other times. The explanation of this fact was given by Stokes, who pointed out that the differential movement of the air in wind would produce the observed effect.

Let A_0 B_0 (Fig. 15) represent the section of a wave-front—*i.e.* a surface in which the disturbance all started from the source at the same instant, and let us suppose that this wave-front at the given instant is vertical and is travelling horizontally. Let A_1 B_1, A_2 B_2, &c., represent successive positions of the same wave-front after equal intervals of time, when the air is quite still.

If the wind is blowing with the sound, then the velocity of the wind must be added to that of the sound. But the upper layers of air move more quickly than the lower, so that the upper parts of the wave travel more quickly than the lower, and the wave-front turns downwards, as shown in Fig. 15, *a*. There is, therefore, a condensation of the sound along the surface of the earth, and a listener at O has much more chance of hearing.

If, on the other hand, the wind is blowing against the sound, its velocity must be subtracted. Then, as in Fig. 15, *b*, the lower parts of the wave travel more rapidly than the upper parts, the wave-front turns upwards, and the sound tends to go into the upper air, and there is a thinning off near the surface, so that a listener at O has much less chance of hearing.

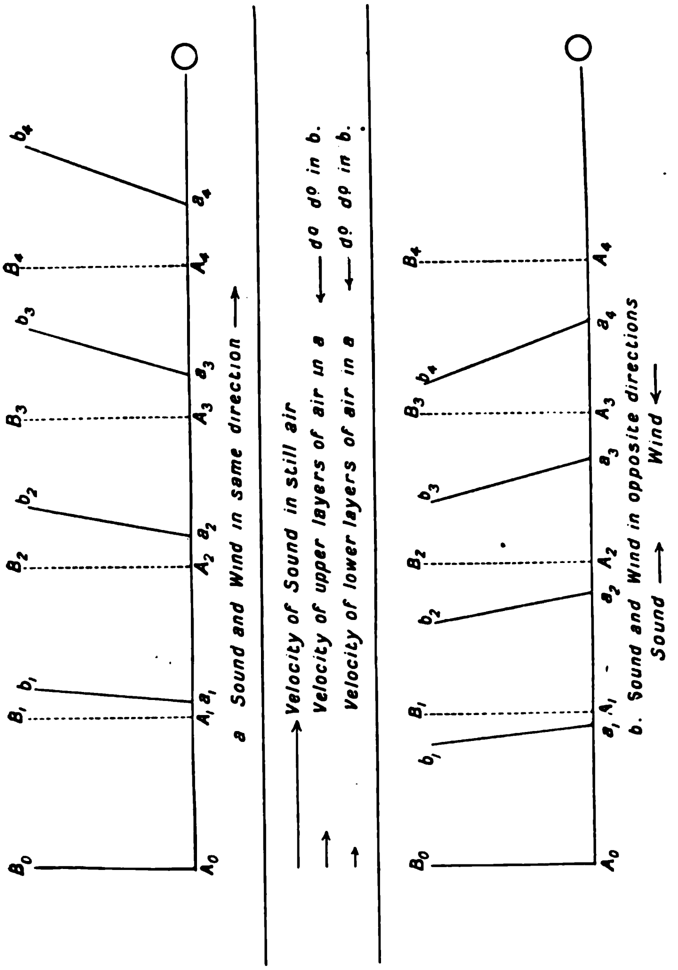
O
B_0
A_0
B_1
b_1
A_1
a_1
B_2
b_2
A_2
a_2
B_3
b_3
A_3
a_3
B_4
b_4
A_4
a_4
a. Sound and Wind in same direction →
Velocity of Sound in still air
Velocity of upper layers of air in a ← do do in b.
Velocity of lower layers of air in a ← do do in b.
b. Sound and Wind in opposite directions
Sound →
Wind ←

FIG. 15.—Refraction of Sound by the differential motion of the successive layers of air in wind. A_0 B_0, A_1 B_1, &c., successive positions of wave-front in still air; a_1, b_1, a_2, b_2, &c., their positions in moving air.

Refraction by Air of Varying Temperature.—Reynolds has shown that there is another kind of sound-refraction, due to temperature variation at different altitudes. If the temperature were the same at all levels, then the velocity of sound would be the same at all levels, since it is independent of the pressure. But if, as is almost always the case, the temperature varies, the velocity varies with it by about $\frac{1}{546}$th of the whole amount per 1° C.

Usually in the daytime there is a fall of temperature as we go upwards, and the velocity is therefore greatest near the ground; the effect is like that represented in Fig. 15, *b*, the waves turning upwards. The sounds thin out along the ground and are concentrated overhead, where there is no observer to note them. If the temperature rises upwards, the effect is the reverse, like that represented in Fig. 15, *a*; the waves turning down, so that the sound is heard more plainly along the ground. This often happens about sunset, when the air below cools more rapidly than that above, and ultimately becomes colder than the higher air, as is shown by the mists rising for a few feet. Then sounds carry along the level to very great distances. Probably every one is familiar with the clearness with which sounds are heard across still water at dusk. On a frosty morning, too, after a very clear night, sounds carry exceedingly well for a similar reason. The contrary effect, due to a lower temperature above, is very noticeable on a hot summer-day, when sounds carry only short distances along the level. But here the apparent stifling of the sound may be aided in another way. If the fall of temperature upwards is sufficiently rapid, the air becomes unstable and up-and-down convection currents are formed. The air is thus a heterogeneous mixture as regards temperature, and the waves of sound are being continually reflected and refracted at the surfaces separating the different currents. The waves are not absorbed, but they are broken up and spread out. Light is affected in a similar manner, and the state of the air may be recognised by the dimness of outline of objects. Tyndall has illustrated the effect of heterogeneity on sound by showing that the sound from a reed, which will ordinarily affect a sensitive flame placed near it, will no longer do so when the hot gas rising from a number of burners is interposed.

It is very probable that the long rolling of thunder from distant lightning is a case of reflection and refraction at the surfaces separating masses of air in different conditions, the more or less sudden sound which we have from a near flash being gradually drawn out into a long roar as it travels on.

Propagation of Sound through Fog.—Lord Kelvin has shown that the limiting condition of equilibrium of the air in a warm fog is reached when the rate of fall of temperature upwards is only about half that in the limiting condition of fog-free air. Hence foggy air is in a more homogeneous state than ordinary air, and we might expect it to carry sound well. This is entirely borne out by Tyndall's observations.[1] If, then, the roll of thunder is due to reflection by the clouds, it is not that they are opaque to sound, but merely that they are in a different acoustic state from the surrounding air.

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[1] *Sound*, Lecture vii.

CHAPTER III.

FREQUENCY AND PITCH OF NOTES.

CONTENTS.—Methods of Determining Frequency of Vibration of Sources—Graphic Methods of Determining Vibrations—Stroboscopic Methods—Manometric Flames—Alteration of Pitch with Motion of Source or Observer—Musical Scale and Temperament.

WE shall now describe some of the various methods which have been devised to determine the frequency or number of vibrations per second of a source emitting a note of given pitch, selecting methods which appear most important, either historically or practically.

General Principle of Methods of Determination.—Usually a note nearly the same in pitch as that to be determined is sounded by some mechanism whose frequency can be counted. When the two notes are sufficiently near in frequency, the experimenter hears "beats," *i.e.* regular variations in the intensity of the sound, the number of beats per second being equal to the difference in the frequencies of the two notes. These beats enable experimenters without exact perceptions of pitch to say when the notes are near each other, and to determine their relation. Thus, if the known source has frequency n, and if the frequency of the beats is b, then the frequency to be determined is $n \pm b$, the sign to be decided by the circumstances of the experiment.

Savart's Toothed Wheels.—The simplest apparatus for the determination of pitch consists of a toothed wheel, which can be rapidly rotated while a card or thin metal plate is held against the teeth. As each tooth passes, the card is lifted and sends out a wave, and the succession of waves gives a musical note. The speed of rotation is adjusted by trial, so that the note sounded is in unison with that to be determined, and then, maintaining the speed constant, the number of revolutions N in time t is counted. If n is the number of teeth on the wheel, the frequency is $\frac{\mathrm{N}n}{t}$.

Seebeck's Siren.—This consists of a plate, now usually of metal (though it is still called the "cardboard" siren), furnished with concentric circular rows of equidistant holes, and mounted on an axis which can be rapidly revolved. When the plate is revolving sufficiently rapidly, a stream of air from a blower is directed by a nozzle against one of the circles of holes. As each hole passes the nozzle, a puff of air rushes through and sends out a wave. The succession of waves produces a musical note, which must be tuned to the given note. This may be done more readily by fixing behind the rotating disc a "resonator," *i.e.* a vessel shaped so as to sound the given note when blown into. The air in this is set vibrating sympathetically by the waves issuing through the holes, and sends out a loud and clear note when the disc is going at the right speed. If an attached counter shows N revolutions in t seconds, and if the holes in the circle are n in number, then the frequency of the note sounded is as with the toothed wheels

$$Nn/t.$$

6

A combination of these two instruments, made for illustration of general principles, is shown in Fig. 5, Chapter I. Neither would be now used for any exact experiment.

Cagniard de la Tour's Siren.—A common form of this instrument is presented in Fig. 16. A small horizontal circular disc, ss, with a circular row of holes in it, is mounted on a vertical axis, so that it turns close to the top, but just clear of a hollow box, A, having a similar row of holes in its top cover. Air is blown into the box from a bellows, and when the holes in the disc coincide with those in the box, a puff issues through each. These separate puffs all coincide to give one big wave. Each time, therefore, that a hole in the disc moves one place on, a wave is sent out; and if n is the number of holes, and N the number of revolutions in t seconds, Nn/t is the frequency of the note sounded. A screw thread, t, on the spindle turns the mechanism which counts N. By cutting the holes in the box and disc slanting in opposite ways, as shown in the section in the lower right-hand side of Fig. 16, taken through *nn* in the upper right-hand figure, the stream of air is made to drive the instrument as well as to give the note. This device was adopted by the inventor, and has been followed since by instrument-makers, but it detracts very much from the exactness of the instrument. With the air blast as motive power it is exceedingly difficult to maintain a constant speed, the tendency being to a gradual increase.

The counter, too, in the ordinary arrangement with an impulse on the slow wheel after every revolution of the quick wheel, is objectionable as making the running jerky. Helmholtz, who used a modified form of the instrument in his celebrated researches, drove it by an electric motor, and used the stream of air merely as a sounder. No doubt this plan would be always followed, now that good electro motors are so easily obtained, if the instrument were required for exact work.

As a matter of curiosity, we must mention the fact that the siren sounds under water if entirely immersed and driven by a stream of water.

Dove modified the instrument by having several circles of holes in the disc, any one of which can be used at will. Helmholtz[1] used a double siren with this modification, in which two discs were mounted on the same vertical spindle with one wind-chest below the lower disc, and the other above the upper disc. The upper wind-chest could be turned about the axis into any required position by means of a pinion gearing with a toothed wheel attached to the top of the chest. By this arrangement, if the two sirens were sounding the same note, the difference of phase could be made zero or anything desired by making the opening of the upper holes coincide with those of the lower holes,

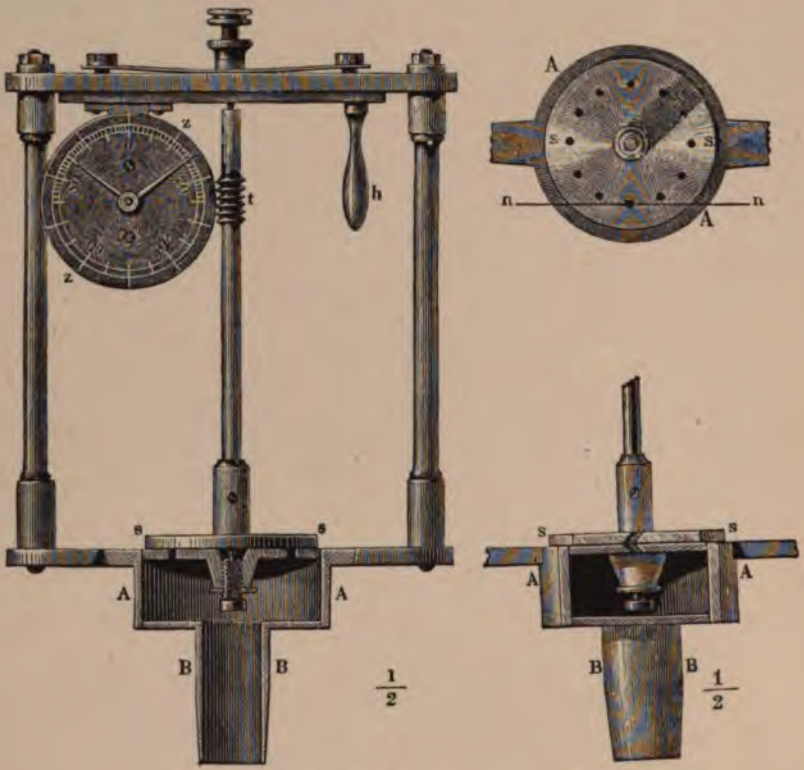

Fig. 16.—Cagniard de la Tour's Siren, about ½ scale.

A. Wind-chest perforated with a circle of holes at the top; s, rotating disc perforated with an equal circle of holes. The slope of the holes in chest and disc is shown in the lower right-hand figure, which is a section through *n n* in the upper right-hand figure.

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[1] *Sensations of Tone*, 1st Eng. ed., p. 243.

or lag behind by the proper amount. The air stream was produced by a stiff paper turbine attached to the disc. Helmholtz succeeded in producing extremely constant notes with this siren, and with a good device for recording the number of revolutions in a given time, it would probably be as good as any other instrument for the determination of absolute frequency.

Scheibler's Tonometer.—This consists of a series of tuning-forks spread over an exact octave and ascending by equal steps in frequency from the lowest to the highest. The steps are so short that the beats between two consecutive forks can be easily counted. When the series is exactly adjusted the number of beats between all the consecutive pairs is the same. It is then easy to calculate the pitch of any one fork. For suppose that there are in all sixty-five forks, and that the lowest makes n vibrations per second; the highest, being its octave, makes $2n$ vibrations per second. The adjustment of these two with each other directly is possible, since, for a reason to be given later (Chapter X., Combination Tones, p. 155), any inaccuracy in tuning would lead to beats of frequency equal to the difference between $2n$ and the frequency of the higher fork. Let the steps in frequency of the intermediate forks be four per second, each adjusted exactly by means of the beats. This can be done, since four beats per second can be counted easily for a long time. Then the series of forks, which we may represent by

$$0 \quad 1 \quad 2 \quad 3 \quad 4 \quad . \quad . \quad . \quad . \quad . \quad 64$$

will have frequencies

$$n, \ n+4, n+2\times 4, n+3\times 4 \quad . \quad . \quad . \quad . \quad . \quad n+64\times 4$$

But No. 64 has frequency $2n$.

So that $$n+4\times 64=2n,$$

Hence and $$\left.\begin{aligned} n&=256 \\ 2n&=512 \end{aligned}\right\}.$$

Thus the frequency of every fork is known.

The tonometer, though extremely troublesome to adjust, is, when once made, not only an exceedingly accurate, but also an exceedingly constant register of absolute frequency, for the physical properties of forks remain practically constant. The effect of temperature change is the only variable element, and this is only slight, being, according to M'Leod and Clarke, a decrease of frequency of about ·00011 per 1° rise in temperature.

Of course, any other source of frequency within the range of the tonometer may be determined by counting its beating frequency with the fork nearest to it. If all the forks are available, it is easy to find whether it is sharp or flat, and therefore whether the beats are to be added to or subtracted from the frequency of the fork. For, if sharp, it will beat more with the lower than with the higher neighbour of the given fork; if flat, the reverse will hold. If there is only the one fork available, then either the frequency of the fork or that of the source must be slightly altered. In the case of the fork, this may be done by loading one prong with wax. The effect on the beats is obviously opposite in the two cases, reducing them if the source is flat, and increasing them if it is sharp.

Appunn substituted free harmonium reeds for forks to form a tonometer, but with a loss of accuracy; for Lord Rayleigh has shown that the consecutive reeds affect each other's frequencies, so that a reed has different frequencies when sounding with each of its two neighbours. He was thus able to explain certain discrepancies found by Ellis between the stated frequencies of some forks made by Koenig and those as determined by the reed tonometer.[1]

GRAPHIC METHOD OF DETERMINING FREQUENCY.

In this method either a vibrating source or a receiver affected by waves from it is made to write its vibrations on a prepared moving surface. The simplest device is illustrated by Fig. 17, where a light style affixed to one prong of a fork writes its vibrations on the surface of a drum covered with smoked paper.

To measure the number of vibrations in a given time, the fork and drum may form part of the secondary circuit of an induction coil, and through the primary of this short currents may be sent by a clock, which makes the circuit at definite known intervals. There will then be a spark from the style to the drum, which will mark the paper at each signal, and the number of vibrations between two successive marks may be counted. Another method of marking known intervals consists in placing a second style level with the first and working it by the currents from the clock.

The source to be determined is to be sounded with the fork and the difference of frequencies determined by the beats.

This arrangement of fork and drum is really more often used as a chronograph, *i.e.* as an instrument for measuring intervals of time, assuming the frequency of the fork to be known. For instance, suppose that it is desired to know the exact time of fall of a body through a certain height, it may be made to break a circuit at the start and again at the arrival at the lowest point. If this circuit is the primary of a coil, the interruptions of current may be made to give sparks from style to drum; then, on counting the vibrations between the marks made thereby, the time of fall is known.

Electric Maintenance of Tuning-Forks.—For chronographic and other purposes it is often necessary to keep a fork, such as that used with the revolving drum, in vibration for a long time together. The general principle of electric maintenance will be, perhaps, most easily understood from a description of the mode of action of a fork used by Helmholtz, and represented diagrammatically in Fig. 18.

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[1] *Nature*, xvii. 1877, p. 13.

If the fork is held in position, it will be seen that the circuit is completed through the upper prong of the fork, the mercury cup, and the electro-magnet coils. But the poles N. and S. tend to draw the prongs apart, and if the fork is released, they fly outwards; contact is broken in the cup, and the poles are demagnetised: the prongs fly back, make contact again, and so on. During the outward motion of the prongs, the magnet is doing work on the fork, and during the inward motion the fork is doing work on the magnet. If these quantities were equal on the whole, no energy would be drawn from the electrical system, and the vibrations would die away. But for two reasons, the former quantity of work is the greater, viz. (1) that the break of contact is delayed by the adhesion of the mercury to the platinum tip of the wire, while the make is delayed as the point does not at once break through the film; and (2) that the effect of self-induction is to delay the full magnetisation at make during the motion inwards, and to sustain the magnetisation by the

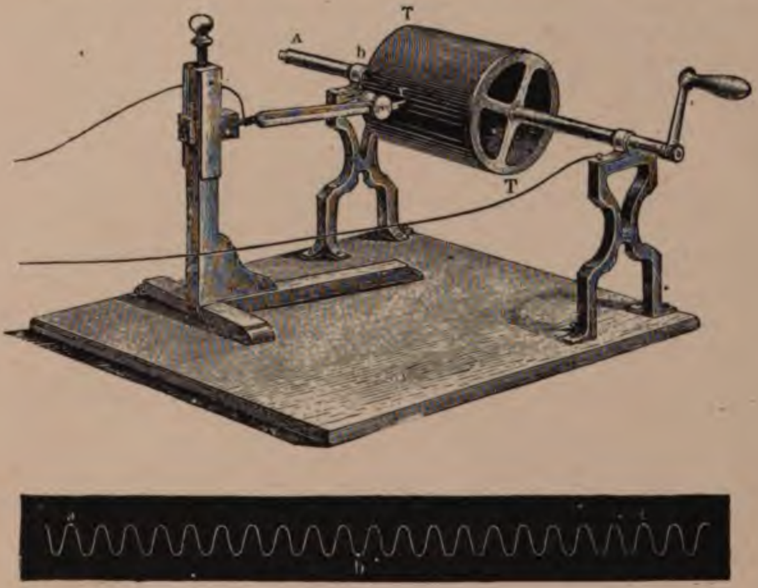

FIG. 17.—Tuning-Fork recording its vibrations on a revolving drum. a, b, c, spark marks on the record.

extra current at break during the motion outwards. The balance of work done by, over work done against the magnet is available

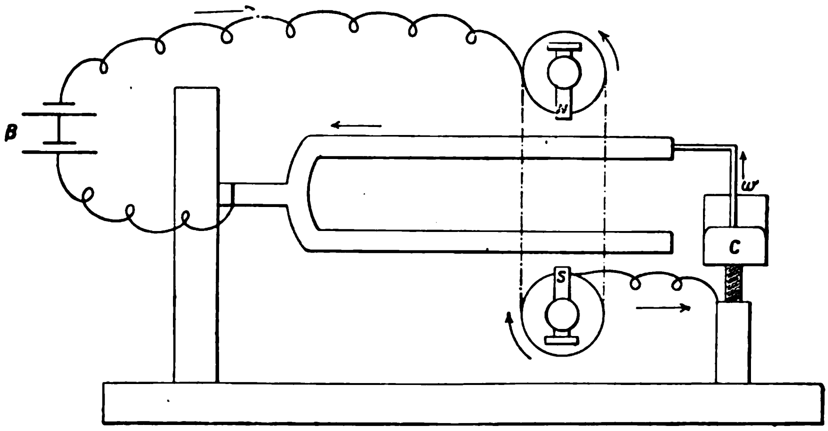

Fig. 18.—Diagram of Helmholtz's Electrically-maintained Tuning-Fork, generally used as an Interrupter. N S, adjustable pole-pieces of an electro-magnet, excited by a current from battery B, which passes up through the mercury cup C, the wire *w*, and the upper prong of the fork back to the battery. The wire *w* just dips into the mercury, so that the circuit is broken when the electro-magnet is excited.

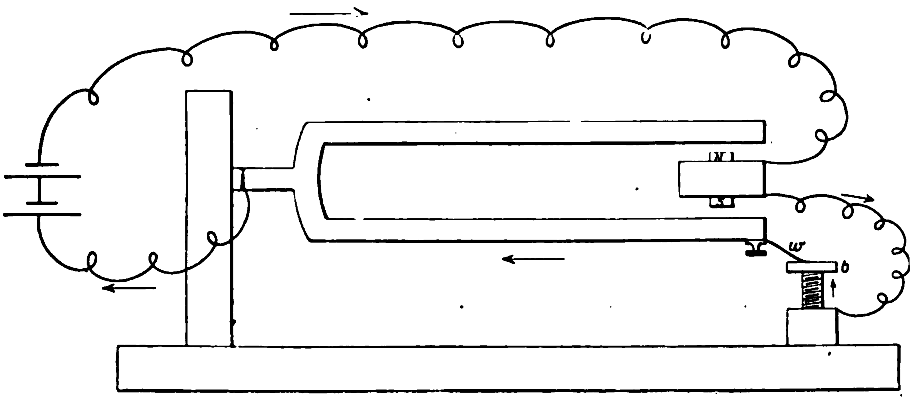

Fig. 19.—Common Form of Electrically-maintained Fork. The wire *w* just touches the contact block *b*, when the electro-magnet N S is unexcited. If the wire *w* is removed, and an interrupted current of nearly the frequency of the fork is sent through the electro-magnet, the fork is kept in vibration with the frequency of the current.

for the energy of sound radiated out by the fork and the energy dissipated against frictional and viscous forces.

Another common form of electric maintenance is represented by Fig. 19, where the electro-magnet is placed between the prongs, and the contact breaker is a platinum wire pressing lightly against an adjustable contact block. The self-induction is here again, doubtless, aided in increasing the balance of energy supplied over energy withdrawn, by the manner in which the wire makes contact.

If the interrupted current from a fork, like either of those just described, be sent through the coil of an electro-magnet placed, as in Fig. 19, between the prongs of a fork of nearly the same period (the second fork now having no contact breaker), then this fork will be set in vibration, not in its own period, but in that of the interrupting fork; and the nearer the two are in frequency, the greater is the vibration of the second fork. But it is not necessary that the second fork shall be nearly of the same frequency. It is sufficient that it is very near to one of the multiples of the frequency of the interrupter. For, as we shall see in Chapter V., a vibration of given period may, in general, be regarded as a compound, made up of a number of simple constituents or "harmonics;" the first, of the actual period and frequency; the second, of half the period and twice the frequency; the third, of one-third the period and thrice the frequency; and so on. If the dependent fork is very near in period to any one of these harmonics which is prominent in the vibration of the interrupter, it will vibrate in the period of that harmonic.

Stroboscopic Methods.—There is an old experiment in which a rapidly revolving wheel is made to appear at rest, if viewed only by intermittent flashes of light, so timed that during each interval of darkness one spoke moves exactly into the position of the next. If the wheel goes at rather less than this speed, it appears to travel slowly back; and if at rather more, it appears to travel slowly forward; for, in the first case, each spoke just falls short of the position of the next for which it is mistaken, and in the second case it just exceeds it. This principle has been adopted in several forms to determine frequency.

The Experiments of M'Leod and Clarke.[1]—The tuning-fork to be tested is fixed with its prongs (Fig. 20) between a revolving drum and a microscope. Lengthwise on the drum are ruled equidistant white lines, and an image of these is thrown, by a short focus lens, into a plane in which the fork is fixed. The image of the lines as interrupted by the fork is viewed by the microscope. When both drum and fork are at rest, the appearance is as represented in Fig. 21, A. If the drum is now revolved, the right-hand side of the field will appear grey. If the fork is also set vibrating, *and if the time of one vibration is just equal to the time taken by one line to move into the place of the next*, the division between the two parts of the field will appear waved, as in Fig. 21, B, for whenever a white line is in a given position, the prong of the fork is in a definite position, and cuts off a definite part of it,—most when the prong is at one end, and least when it is at the other end of its swing. If the drum turns at rather less than this speed, the white lines have not moved quite so far when the same amount is cut off, and the waves appear to travel slowly backward. If the drum is too

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[1] *Phil. Trans.*, 1880, vol. 171, p. 1.

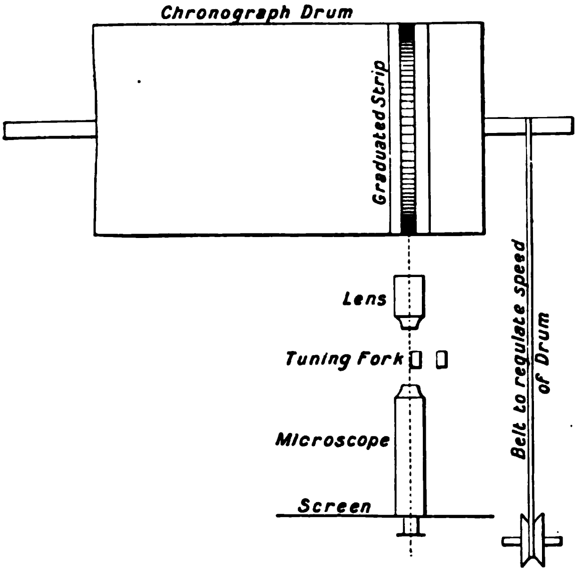

FIG. 20.—Diagram of M'Leod and Clarke's Apparatus for Determining the Frequency of a Fork.

A revolving drum, having a graduated strip round it ruled in white lines, revolves at a known rate. A lens throws an image of the strip at the side of a vibrating fork. The image is examined by the microscope.

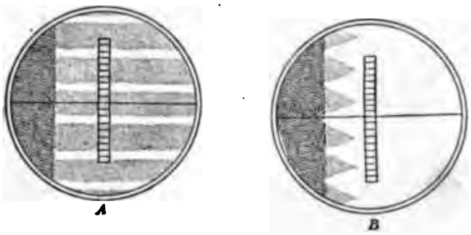

FIG. 21.—Appearance in Microscope. A, when fork and drum are at rest; B, when fork is vibrating in time taken by each line to pass into position of next.

rapid, the motion appears to be in the direction of rotation. The speed of the drum is under control, and a counter gives the total number of revolutions during a time exactly determined by electric signals from a pendulum. When the waves are quite stationary, the number of lines passing the centre of the field in any time is equal to the number of vibrations of the fork in the same time. If the waves are not stationary, the number of waves passing must be added or subtracted according to their direction of motion.

Lord Rayleigh's Method.[1]—If two light metal plates are so fixed on to the ends of the prongs of a fork that at the extremity of the outward swing they just draw apart and leave a gap, an eye looking through the gap will have a view really made up of a rapid succession of views, one for each vibration of the fork. By electric maintenance the swing may be kept of constant amplitude, so as to make the gap every swing. If the eye looks at a pendulum through the gap, the eye will see as many positions of the pendulum as there are vibrations of the fork in one vibration of the pendulum. If the period of one is an exact multiple of that of the other, the positions will appear stationary; but if the accordance is not exact, the position will appear to change slowly, moving forward if the pendulum gains, backward if it loses. This was adopted as the principle of a method devised by Lord Rayleigh to determine exactly the frequency of a certain fork marked as having frequency 128, the correctness of the marking being open to doubt.

A fork of frequency about 125 was electrically maintained by an interrupter fork of frequency about 12½. On this last the thin metal plates were fixed to form the intermittent gap, and a seconds pendulum (making one complete vibration in two seconds), having an illuminated bead on it, was viewed through the gap; there were, therefore, twenty-five positions of the bead. To enable one of these positions to be studied, a narrow vertical slit was fixed in front of it. The position of the bright spot at successive flashes appeared to move slowly back across the slit, showing that the pendulum lost on the fork. After a short time the bright spot disappeared altogether, and eighty seconds from its first appearance the next position had retreated so as to come into view. It passed across the slit, disappeared, and so on every eighty seconds. The fork, therefore, made one more than 80 × 12½, or 1001 vibrations in eighty seconds. This gave a frequency of 12·5125. The dependent fork, having naturally very nearly ten times this frequency, was made to vibrate 125·125 times per second. The fork to be tested made with it 180 beats in sixty seconds, or three beats per second, whence its frequency was 128·1. A further development of the method is described in the *Philosophical Transactions*.[2]

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[1] *Nature*, xvii. p. 12.

[2] Pt. i., 1883, p. 316.

Koenig's Manometric Flames.—Manometric flames, or flames showing variations of pressure, are especially suitable for comparing the relative frequencies of pipes. A manometric flame is

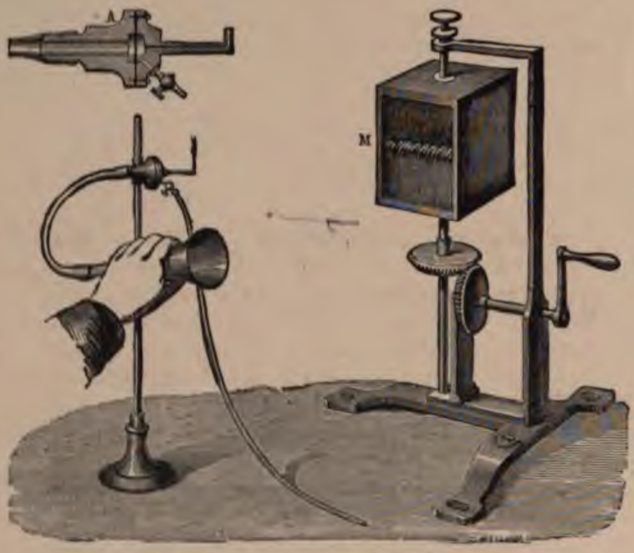

FIG. 22.—Koenig's Manometric Flame.

arranged as shown in Fig. 22, A. The gas, on its way to a pinhole burner, passes through a small chamber closed on one side by a membrane three or four centimetres across. If there are rapid variations of pressure on the left-hand side of the membrane, it moves in and out, and checks or aids the flow of gas to the burner. The flame is thus made variable, and jumps up and down with a frequency the same as that of the membrane. If, for instance, a note be sung into a mouthpiece connected with the left-hand compartment, the flame is affected. But in general the vibrations are too rapid to be seen separately, and the only direct indication of their existence consists in the peculiar drawn-out appearance of the flame, as shown in Fig. 23, where *a* shows its shape with steady pressure and *b* its shape when jumping up and down. If the eye be rapidly carried past the flame,

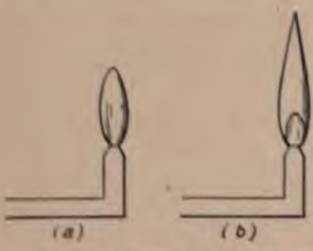

FIG. 23.—Manometric Flame. (*a*) pressure steady; (*b*) pressure intermittent.

its intermittent nature is at once revealed by a jagged or toothed appearance. To show this properly, a cubical box (Fig. 22, M) with a plane mirror on each of its four vertical sides is rotated rapidly about its vertical axis, and the reflection is viewed in the mirrors. A band of light is then seen, continuous when the flame is continuous, jagged or toothed (as in Fig. 24) when it is intermittent. This figure, given by Koenig, shows the effect of sounding vowels into the mouthpiece pitched on different notes. For comparing the frequencies of two pipes or other sources, two manometric flames may be placed close together, affected respectively by the two sources, and their reflection compared, the number of teeth in the same distance on each being counted.

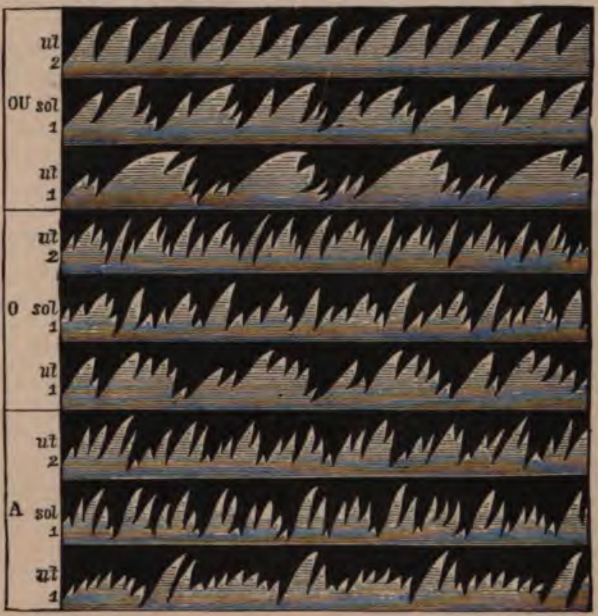

FIG. 24.—Flame-Pictures of the Vowels OU, O, and A (*Koenig*).

Lissajous' Method of Comparing Forks.—This method consists in studying the curves formed by the composition of the two vibrations to be compared when they are arranged at right angles. We only mention it here as a possible method, but we shall describe it later (p. 77) when we have discussed the subject of the composition of vibrations.

In concluding the subject of determination of pitch, it may be worth while to mention a method of determining very roughly the frequency of a source when that of another, say a fork, is known. This consists in tuning a monochord or sonometer (p. 81), so that a length l is in unison with the source of known frequency n, and then adjusting the bridge till another length, l', is in unison with the source of which the frequency n' is to be determined. Then, since $n'/n = l'/l$, n' can at once be found.

ALTERATION OF PITCH WITH MOTION OF THE SOURCE, OBSERVER, OR MEDIUM.

Döppler's Principle.—The commonest example of this kind of alteration of pitch is the lowering of the note heard from the whistle of a locomotive as it passes the observer. The explanation of this was first given by Döppler, whose name is attached to the principle on which the explanation is founded. As the engine approaches, it follows up the waves which it has first sent out in the direction of the observer, and so crowds a greater number into a given length of air than if it were at rest. The number received per second is greater than the number sent out, and so the pitch rises.

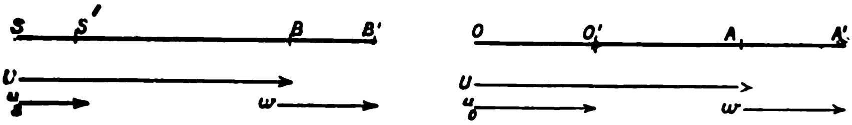

FIG. 25.—Alteration of Pitch by Motion of Source, Observer, or Medium. S and O, initial positions of source and observer; S'O', their positions after one second; $OA = SB = U$, velocity of sound in still air; $OA' = SB' = U + w$, velocity when wind is blowing with velocity w; $OO' = u_o =$ velocity of observer; $SS' = u_s =$ velocity of source.

When, on the other hand, the engine is receding, it draws away from the waves which it first sent out to the observer after passing him, and so sends fewer into a given length than if it were at rest, and the pitch falls.

In obtaining an exact formula, which gives also the effect of motion of observer and of wind, we shall first find the length of air which gives up its waves to the observer in one second, and then we shall find the number of waves put into this length by the source, if the number it emits per second is n.

Referring to Fig. 25, the wave which reaches O at the beginning of a second, is at A', a distance $U + w$ from O, at the end of the second, for it travels U relatively to the air, and the air travels w relatively to the ground. Meanwhile, O has moved on to O', a distance u_o forward, so that the waves in length $O'A' = U + w - u_o$ have been received by the observer.

Now, turning to the source, the wave which it emits at S at the beginning of a second is at the end of that second at B', a distance $U + w$ from S. Meanwhile, S has moved on to S', a distance u_s forward, so that the n waves emitted in the second are contained in distance $S'B' = U + w - u_s$. Then the number contained in distance $U + w - u_0$ will be

$$n.\frac{U + w - u_0}{U + w - u_s},$$

and this is the frequency of the note heard by the observer. If both source and observer are at rest, or are moving with the same velocity, then the pitch is unaltered, for numerator and denominator are the same. If the observer and source only move slowly as compared with $U + w$, it is evident that a motion of the source has the same effect as the equal and opposite motion of the observer. But this ceases to hold for large velocities, as may be seen by putting $w = 0$, and then in turn making $u_s = 0$, $u_0 = \frac{U}{2}$, and $u_0 = 0$, $u_s = \frac{U}{2}$. In the first case the lowering is an octave, and in the second a fifth. The consequences of making u_0 or u_s greater than U are curious rather than practically important, and may be left to the reader to follow out.

It will give an idea of the order of the effect if we calculate the speed at which two trains must be going in order that the pitch of the whistle of one as heard on the other may change by a whole tone, *i.e.* in the ratio 9 : 8 when they pass.

If n is the frequency of the whistle and $\pm u$ the velocity of either train, the frequency of the note heard on approach is

$$n.\frac{U + u}{U - u},$$

while that of the note heard on drawing apart is

$$n.\frac{U - u}{U + u}.$$

Putting

$$n.\frac{U + u}{U - u} = \frac{9}{8}n\frac{U - u}{U + u},$$

$$8(U + u)^2 = 9(U - u)^2,$$

whence, if

$$U = 1100 \text{ ft./sec.}$$

$$u = 32 \text{ ft./sec.}$$

or say 22 miles an hour.

Döppler's principle has been verified in open-air experiments on railways by Buys-Ballot, Scott-Russell, and Vogel. Various laboratory experiments have also been devised to show the effect of motion of the source, of which the simplest is due to Mach. A whistle, or better still, a pitch-pipe, is put into the end of an india-rubber tube 5 or 6 feet long. The experimenter whirls this in a horizontal circle round his head, at the same time blowing into the open end. An observer hears most distinct rises and falls in pitch as the source approaches and recedes. Koenig illustrated the principle by two forks slightly out of unison, and making, say, four beats per second. On sounding these together and bringing the sharper towards the observer, the beats increased in frequency, while on taking it away they diminished.

The subject is rather one of curiosity than of importance in Sound, but in Light its application to the light received from the sun and stars, and analysed by the spectroscope, has enabled us to determine their motion relative to the earth, and has indirectly given us knowledge as to the constitution of the stellar system, which we could hardly hope ever to reach by direct observation.

Limits of Audibility.—In order that a series of waves striking on the ear may excite the sensation of musical sound, their frequencies must be neither too great nor too small, lying between limits which depend on the individual. When the upper limit is passed, the ear is unaware of the existence of sound; and the variation in the position of the limit with different individuals is sometimes most strikingly illustrated when the air appears to one observer full of high sounds due to the chirping of grasshoppers, when to another with him all is silent. Accordingly, different experimenters have found very different limiting frequencies. Savart, with a toothed wheel, heard a note of 24,000. Despretz could hear a small tuning-fork of 36,864, and Appunn one of 40,960. Koenig constructed a harmonicon with a series of bars vibrating, when struck, with calculable frequencies, and he found that the range varied with age, older persons not hearing beyond 16,384, while no one could hear 24,576.

At the lower limit the ear may begin to perceive the separate impulses, but there is great difficulty in determining where the sense of continuity ceases. The note sounded is always a compound; the lowest or fundamental tone, the one to which attention should alone be directed, being accompanied by a number of higher tones, the "overtones," which the observer may perceive and mistake for the fundamental. He is the more likely to do this as the sensation of sound is much more easily excited by a given amount of energy received per second in the form of short than of long waves; *i.e.* a weak overtone may be more conspicuous than an energetic fundamental. Helmholtz made experiments in which the source was a string loaded in the middle so that the first overtones were several octaves above the fundamental, and not liable to be mistaken for it. When the frequency was 37 there appeared to be a sound, but at 34 there was scarcely anything audible. Tuning-forks making swings with an amplitude of 9 mm. gave a weak drone at 30 vibrations per second, but could hardly be heard at 28. From these experiments, which are far more trustworthy than any earlier ones, we may conclude that the lower limit is not far from 30 vibrations per second.

The Number of Waves in Succession Needed to Give the Sensation of Tone.—This, like the preceding, is of physiological rather than physical interest. Pfaundler[1] experimented on the subject by arranging a "cardboard siren" with six holes. This was rotated 10 times per second. Instead of a single nozzle, two were fixed close to each other, so that a hole passed from one to the other in $\frac{1}{720}$ of a second. Thus 60 times per second two waves were sent out at an interval of $\frac{1}{720}$ second, and a note of frequency 720 could be perceived, whence it was concluded that two impulses in succession are sufficient to give rise to the perception of tone. Herroun and Yeo[2] arrived at the same conclusion from a similar experiment.

Another observation seems to verify this. If an observer stations himself near a wall on which are impinging the waves from some such source as a waterfall or a railway train, each impulse enters the ear twice, first directly, then as a reflection; and the observer accordingly hears a tone of wave-length equal to twice the distance between himself and the wall. The sense of pitch does not in these cases of two vibrations appear to be very definite, but when the method of experiment is modified, so that the number of successive vibrations rises to 16 or 20, it becomes quite determinate.

THE MUSICAL SCALE.

The musical scale is the succession of notes given, for instance, by the keyboard of a piano. After each succession of seven white keys, forming with the first of the next scale an octave, the intervals are all repeated in the same order, and the notes are signified by the same letters. It is usual in Sound to use different type for the different octaves, and in Helmholtz's notation the octave from bass to middle *c* is written

$$c\ d\ e\ f\ g\ a\ b\ c'$$

The octave above is accented thus:

$$c'\ d'\ .\ \ .\ \ .\ c''$$

The next octave has two accents, and so on.
The octave below bass *c* is written

C D E F G A B *c*

The octave below this

$$C_{,}\ D_{,}\ E_{,}\ F_{,}\ G_{,}\ A_{,}\ B_{,}\ C$$

and each succeeding octave has another accent added as suffix.

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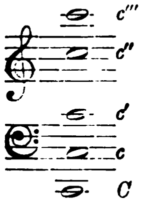
[1] Muller-Pouillet, 1886, i. 731.

[2] *Proc. Roy. Soc.*, l. No. 305, p. 318.

We have then



The investigation of the physical relations of the various notes of the scale, and of the physical qualities corrresponding to recognised musical effects, is of the greatest interest and importance. We owe to v. Helmholtz[1] the most thorough inquiry which has yet been made on the subject, and the reader who desires more extended knowledge is referred to his work. Here we only give a very brief account of the beginning of the subject.

It has already been stated (Chapter I. p. 10) that two notes forming a definite musical interval have their frequencies in a fixed ratio in whatever part of the scale the two notes are situated. Thus the upper note in an octave has twice the frequency of the lower; in a fifth it has 3/2 the frequency, and so on. The musical scale may, therefore, be discussed to a large extent without reference to absolute frequency, the ratios being often sufficient. Nevertheless, each different note has a more or less definite frequency, or rather range of frequency, though in the course of time the frequency has shown a tendency to rise. Thus or c'' appears early in the last century to have had a frequency below 500, and to have risen to 500 or 512 by the time of Handel. As it continued to rise, *i.e.* as there was a tendency to tune instruments higher and higher, attempts were made to fix the pitch. A Congress at Stuttgart in 1834 proposed 528, and a French Congress in 1859 proposed 522. In England at the present time, "concert pitch" gives the note a frequency of 546.

But the forks made by Koenig for laboratory use are all tuned to the scale c'' or $\mathrm{Ut}_4=512$ (marked, in accordance with the French plan, 1024, the number of half vibrations), and this is the standard adopted universally for scientific purposes.

It may here be noted that the human voice ranges over some three or four octaves, the speaking voice, which is lower than the singing voice, being, for men, usually somewhere in the octave below c; while the singing voice, for women and children, extends into the octave above c''.

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[1] *Sensations of Tone.*

The Diatonic Scale.—We are so accustomed from the earliest childhood to the diatonic scale, *i.e.* the common succession of eight notes forming an octave in the major mode, either through hearing it on pianos and other instruments or on peals of bells, that we are hardly inclined to regard our sense of its fitness as a result of education. Rather, we feel that had we never heard it, we should instinctively form it for ourselves if we attempted a succession of notes. But there is no doubt that this scale is, to each one of us, a matter of education, and to the race a matter of slow growth. It is one out of many used at different times by different peoples, and finally adopted by European nations from its fitness to develop the style of music cultivated by them. We are still familiar with another scale, that used in the old Scotch airs, and represented nearly by the black notes on the piano. "Auld Lang Syne" and "The Campbells are Coming" will serve as examples of the use of this scale. The reason for the selection of our present scale is still uncertain, but it appears very probable that it is somewhat as follows.

The music which has in course of time developed into our present European music has always been characterised by a "key-note," or one particular note to which all the others in the melody are referred, or with which they make quite definite intervals. There is never a continuity of tone from one note to the next. Such continuity, as in the whistling of the wind, or in the moaning of people or animals in pain, is always disagreeable, perhaps from its association with pain. But this avoidance of continuity and the use of intervals does not necessitate the actual intervals chosen. Were it a question of harmony or the sounding of two notes together, it would be easier to explain the adoption of the particular ones chosen; for a physical explanation can be given for the pleasure or displeasure when two notes are sounded together, in the absence or presence of beats of a particular range of frequency. This is a subject to which we shall return (Chapter X. p. 151). But though the pleasing intervals of melody or succession are the same as those of harmony or coincidence, the scale was developed long before harmony was used in music, at least consciously. It is perhaps possible that the very early use of stringed instruments may have unconsciously introduced harmony through the continuance of one note after the next was struck, but this can hardly be a complete explanation, and we must look for something besides beats, some feature common to harmony and melody, to account for the adoption of the same intervals. The most reasonable explanation which has yet been given is the following, due to von Helmholtz.

Any note sounded by an ordinary musical instrument, including the human voice, but excluding the tuning-fork when mounted on a resonance-box, is accompanied by a number of higher tones or "overtones," usually of twice, thrice, and so on, times the frequency of the lowest or fundamental tone. Those notes which, sounded together, give concords have certain overtones in common, and the nearer the common overtones are to the fundamental, the smoother is the concord. This corresponds to the fact that the beats are then less likely to be within the range of frequency which happens to be annoying to the ear. If the two notes forming a pleasing concord are sounded, not together, but in succession, there are no beats; but still the ear takes pleasure in the common quality, the possession of common overtones, though of course we do not consciously recognise their existence except after special training. The transition from one note to the next most likely appears less abrupt when there is something the same in both, somewhat as, in watching a dance, we are more pleased if a change is made from one evolution to the next by proceeding from some posture common to the two.

Accepting this as the principle on which the scale is founded, we may conveniently use it to derive the scale from our present standpoint rather than by going, in the historical method, through all the steps (many of them failures) taken by our ancestors.

In harmony the concords of two notes within the octave are the following:—

Interval.	Frequency Ratio.	Notes in Key of *c*. Lower.	Higher.
Octave	2 : 1	C	*c*
Fifth	3 : 2	C	G
Fourth	4 : 3	C	F
Major third	5 : 4	C	E
Minor sixth	8 : 5	E	*c*
Minor third	6 : 5	A	*c*
Major sixth	5 : 3	C	A

These give us in all five different notes, C E F G A *c*. The two intervals C—E and A—*c* are much greater than the others, and they may be filled up by starting with G as the keynote and supplying the notes standing to it in the same relations as E and G stand to C. The former is B and the latter *d* in the next octave. Bringing it down an octave gives us D. We then have eight notes, and the frequencies are easily seen to be as under, taking C as the unit—

C	D	E	F	G	A	B	*c*
1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2

This is known as the diatonic scale.

It will be observed that the numbers in the numerators and denominators of the ratios are, in each case, the smallest possible to [illegible] note in the same neighbourhood. Thus, taking C—B, $\frac{15}{8}$, [illegible] ratios with lower numbers are $\frac{14}{8}$ and $\frac{14}{7}$. The latter [illegible] octave C, and the former is nearer to A than to B.

To get any note close to the position occupied by B, then, we should require larger numbers, such as $\frac{31}{16}$ or $\frac{61}{32}$. This smallness of the numbers expressing the ratios is also the condition for the possession of the lowest possible overtones in common, *i.e.* those nearest to the fundamental.

The intervals in the diatonic scales between successive notes are of three kinds, viz.,—

Major tone, viz. C—D, F—G, A—B, $\frac{9}{8}$
Minor tone, ,, D—E, G—A, $\frac{10}{9}$
and Major semi-tone, E—F, B—*c*, $\frac{16}{15}$

As long as we relate all the notes to C, the diatonic scale is sufficient; but frequently the keynote is changed, and then new notes are required, not contained in the scale of C. We shall see this at once if we construct the diatonic scale of G as in the second line of figures in the Table on the next page, where the first line is that of C with the frequencies given for two octaves. We start with C=24 to avoid fractions in that scale.

It will be seen that most of the notes in the scale of G coincide with notes in the scale of C, but that two differ. Near A we have one in the ratio $40\frac{1}{2}/40$ or 81/80 to A. This interval, which is also equal to $\frac{9}{8} \div \frac{10}{9}$, or the difference between a major and a minor tone, is termed a **comma**, and is so small that the note of the C scale may be substituted for that of the G scale without serious annoyance. But near *f* we have a note in the ratio $67\frac{1}{2}/64$ or 135/128 to *f*, and this is an interval which is very perceptible. A new note is therefore necessary, which is termed *f* sharp (*f*♯), and the interval 135/128 is termed a **sharp**. Let us now take as keynotes in succession D, A, E, B, which nearly represent, in the octave C–*c*, the successive fifths from G, viz., 54, 81, $121\frac{1}{2}$, $182\frac{1}{4}$. Forming the diatonic scales starting from these (as shown in the Table), each introduces one more sharp besides those already required, and thus we get one to each of the notes C, D, F, G, A.

If we now begin afresh and go down by fifths from C=24, we have $24 \times \frac{2}{3}$, $24 \times (\frac{2}{3})^2$, $24 \times (\frac{2}{3})^3$, and so on, of which the representatives in the octave C—*c* are 32, $42\frac{2}{3}$, $28\frac{4}{9}$, and so on. Starting with these as keynotes, the first, F, requires a new note, $42\frac{2}{3}$, near B, and at the interval $\frac{128}{135}$ below it. This is termed B flat (B♭). It does not coincide with A sharp, which is $42\frac{3}{16}$, but makes with it an interval of $\frac{2048}{2025}$, which is rather less than a comma. The second, $42\frac{2}{3}$, B♭, is the new note just supplied, and it requires another new note, E♭, at $56\frac{8}{9}$ or $28\frac{4}{9}$. This is the third keynote, and itself introduces A♭. Proceeding thus, we introduce a new flat with each scale. There are also a number of notes which make a comma with those of the original scale. Continuing the formation of the Table, we ultimately obtain, besides the seven notes C D E F G A B, sharps and flats to each, at intervals respectively $\frac{135}{128}$ and $\frac{128}{135}$ from them, and also a number of notes making the small interval of a comma with the original or the supplementary notes. In fact, if we strictly follow the most pleasing melodic intervals, each new keynote introduces new notes, and the number very soon mounts up to a considerable sum. These

TABLE OF SOME DIATONIC SCALES.

KEY-NOTE	C	D	E	F	G	A	B	*c*	*d*	*e*	*f*	*g*	*a*	*b*	*c′*
C	24	27	30	32	36	40	45	48	54	60	64	72	80	90	96
G	...	...	...	...	36	$40\frac{1}{2}$	45	48	54	60	$67\frac{1}{2}$ ♯	72	...	...	...
D	...	27	$30\frac{3}{8}$	$33\frac{3}{4}$ ♯	36	$40\frac{1}{2}$	45	$50\frac{5}{8}$ ♯	54	...	...	...	...	...	...
A	...	...	...	...	...	40	45	50 ♯	$53\frac{1}{3}$	60	$66\frac{2}{3}$ ♯	75 ♯	80	...	...
E	...	..	30	$33\frac{3}{4}$ ♯	$37\frac{1}{2}$ ♯	40	45	50 ♯	$56\frac{1}{4}$ ♯	60	...	...	...	...	..
B	..	...	...	...	...	($42\frac{3}{16}$ ♯)	45	$50\frac{5}{8}$ ♯	$56\frac{1}{4}$ ♯	60	$67\frac{1}{2}$ ♯	75 ♯	$84\frac{3}{8}$ ♯	90	.
F	...	...	...	32	36	40	$42\frac{2}{3}$ ♭	48	$53\frac{1}{3}$	60	64	...	...	...	..
B♭	...	...	...	...	...	...	$42\frac{2}{3}$ ♭	48	$53\frac{1}{3}$	$56\frac{8}{9}$ ♭	64	$71\frac{1}{9}$	80	$85\frac{1}{3}$ ♭	.
E♭	...	...	$28\frac{4}{9}$ ♭	32	$35\frac{5}{9}$	$37\frac{25}{27}$ ♭	$42\frac{2}{3}$ ♭	$47\frac{11}{27}$	$53\frac{1}{3}$	$56\frac{8}{9}$ ♭	...	...	...	...	.

notes are only practicable, on instruments such as the voice or the violin, with a continuous range. On instruments of the keyboard type, where an indefinite multiplication of notes would be cumbrous and mechanically impracticable, a limited series has to serve for all, and a compromise must be made, so that the best representation possible shall be made of the different intervals. This compromise is termed **temperament**.

After attempts at various numbers of notes in the octave ranging from 21 downwards, modern practice has at present settled down to 12; and after various modes of adjustment of these to give as nearly as possible the diatonic scale for favourite keys, the plan now universally adopted really discards the diatonic scale, and divides the octave into 12 equal intervals, each $\boxed{2^{\frac{1}{12}}} = 2^{\frac{1}{12}} = \sqrt[12]{2}$ The convenience of having all the keys equally bad seems to be greater than that of having some better and some worse. Recently, however, there has again been a tendency to use more notes in the octave, so as to approach once more the "just" intonation of the diatonic scale, and several keyed instruments have been devised with this aim. Probably, with the more perfect mechanism now at command, such instruments will be made practical and convenient, and ultimately the equal temperament will give place to a nearer approach to the ideal in which each key is diatonic.

In conclusion, it should be observed that compromise or temperament is not a question merely of instruments and mechanism, but is a necessity in music from the incommensurability of the different intervals. Taking, for instance, the intervals 2, $\frac{3}{2}$, $\frac{5}{4}$, the result of any progression from a keynote by a series of these intervals may be represented by 2^l, $(\frac{3}{2})^m$, $(\frac{5}{4})^n$, where the index of each is the number of times of ascending in excess of those of descending through the interval. If we are to return to the keynote, then the product must equal 1. If l, m, and n, are integers, *i.e.* if we keep to the exact intervals, then $l=0$, $m=0$, and $n=0$, or every step in ascending must be exactly retraced somewhere in descending. But this would be too rigid a rule to follow. Consequently the intervals are slightly altered or tempered, so that, ascending by one set, the keynote may be reached on descending by a different set. As Herschel says,[1] "Any one who should keep on ascending by perfect fifths, and descending by octaves or thirds, would soon find his fundamental pitch grow sharper and sharper till he could at last neither sing nor play; and two violin-players accompanying each other, and arriving at the same note by different intervals, would find a continual want of agreement."

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[1] "Sound," *Encyc. Met.*, § 244.

CHAPTER IV.

RESONANCE AND FORCED OSCILLATIONS.

CONTENTS.—Resonance illustrations by Pendulums, Carriages, Bridges, and Musical Instruments—Forced Vibrations.

EVERY one knows that a gas globe or a wine-glass may be set in vibration in sympathy with its own note sounded from another source. This is an illustration of the very important and general **Principle of Resonance**, of which numerous examples may be found in Mechanics, Light and Electro-Magnetism, as well as in Sound. In all these cases, some body or system having a natural period of vibration of its own, is set vibrating by the vibration of another body or system of the same period. The simplest mechanical example is given by suspending two simple pendulums of the same length (and therefore of the same frequency) from a flexible support, such as a stretched india-rubber cord. A third pendulum of different length should also be suspended for the sake of contrast. If one of the equal pendulums is set swinging, it will gradually share its motion with the other, and, in time, the two will have nearly equal amplitudes. But the third pendulum will never swing through more than a small arc, and careful watching will show that it keeps getting up to a certain amplitude, and then decreasing again to no swing. This gives us the key to the explanation.

Let us call the two equal pendulums, A and B, and the third, C; and let A be set swinging. As it swings to and fro it sends waves of disturbance along the flexible support which pass on to B and C. For simplicity, we shall suppose each wave as equivalent in its action on the pendulum mass to a single impulse, *i.e.* of a force applied in one direction for a short time, there being an interval between the impulses equal, of course, to the time of swing of A. We may think of the impulses as due to the sudden motion to one side of the point of support, and the consequent inclination of the string, so that the tension of the string has for a moment a horizontal component.

Consider first the effect on B. The first impulse disturbs it from rest, and it swings out and back again, and would go on swinging to and fro, the swings decreasing through friction, &c. But when it has just gone through a complete swing, and is again moving as in the first moment, the second impulse arrives, and the motion being in the same direction as that produced by the first impulse, the second impulse does work upon B, *i.e.* gives it energy and increases the swing. This increase goes on with each successive impulse until the amount of energy put in is equal to that sent back, sent out, and lost by air-resistance, &c. Now consider the effect on C, and, for simplicity, let us take the case in which the difference of period between A and C is but small. The first impulse disturbs C, just as it disturbed B, and the mass swings out and back again, and would continue to swing slightly to and fro for a time in lessening arcs. But when it has either nearly completed its swing (if it is longer than A), or has rather more than completed it (if it is shorter), the second impulse arrives; but it is still moving in the original direction, and so it has work done upon it, *i.e.* it receives energy. The swing, therefore, increases. The third impulse arrives during the next swing, when it is again moving in the original direction; but this time it is further from the centre. The successive impulses are received at points further and further from the centre, till at last they come when C is $\frac{1}{4}$ swing away from the centre. After this C is moving in the opposite direction when they arrive, and now they oppose the motion or take energy away from C. The swings, therefore, gradually damp down to zero, and C is at rest. But then the impulses act exactly as before, and the swings begin to grow again, and so on for some little time. Ultimately, as we shall explain later, the pendulum will make a sort of compromise which will enable it to swing in the period of A.

A curious example of the effect of one pendulum on another of nearly the same period has sometimes been observed in the action of two clocks upon each other when fixed against a common and not quite rigid support. One clock has gradually lessened the swings of the pendulum of the other until it has stopped. The explanation is obviously of the kind we have just given for the effect of the pendulum A on the pendulum C.

Many other examples of Mechanical Resonance will be observed when the attention of the reader has once been directed to the subject. Thus he may note that a two-wheeled vehicle often gets up quite a considerable swing when the horse trots at a particular pace, the time of swing of the vehicle then agreeing with the time of oscillation of the horse in trotting; or he may observe the swing of a railway carriage to and fro when its speed is just such as to make the impulses due to the intervals between successive rails synchronise with the natural vibrations of the carriage on its springs. A change of speed at once reduces the swinging.

Many years ago, before the principle was fully understood, or at least supposed to touch practical matters, a suspension-bridge at Manchester broke down when a troop of cavalry was crossing it, the step of the troop just keeping time with the swing of the bridge, and thus making it move beyond the limit of safety. Now bodies of troops, either horse or foot, always break step on crossing such a bridge.

The most striking illustrations of resonance are observed in the case of Sound, and we shall describe in the next chapter the most important application of the principle in the analysis of vibrations. Here we shall mention some simple cases which may be known to the reader, or which he may easily observe for himself.

A familiar case is that of a gas globe or a gong suddenly set vibrating by its own note from another source, say a voice sounding loudly in the same room.

Any hollow vessel, such as a vase with a neck, has a natural note, which may be obtained by blowing gently across the opening. If the vase be held up to the ear—not too closely—and the same note is sounded on a piano or sung, the vase sounds out loudly to the ear against which it is held. By partly closing the opening with a card or with the hand, or by varying the space between the vase and the side of the head, its natural note may be varied, and so it may be tuned to exact resonance with a note to which it would not otherwise respond.

If the pedal of a piano is pressed down so as to free the strings, and a note is sung into the instrument, that note is at once taken up and "echoed" back. This, of course, is really resonance. Further, the note sung into the piano is really a mixture of a number of simple tones which combine to give it its special quality. Each of these tones is taken up by the piano string of the same tone, so that the note returned has much of the quality of that sung into it.

Perhaps the most remarkable case is that of two tuning-forks mounted on resonance-boxes and exactly tuned to unison. If one is bowed in the neighbourhood of the other, and then damped to silence, the second is found to be sounding loudly. Releasing the first, and then, after a short interval, damping the second, the first is found to be sounding, and so on. When the first vibration is large, the energy of vibration may be passed to and fro several times in this way.

FORCED VIBRATION.

Resonance may be regarded as a limiting case of a more general principle, which asserts that if a periodic force be applied to a vibrating system, the system will ultimately vibrate in a period the same as that of the force. Its vibration is then termed forced.

After the first application of the periodic force, some time is required for the system to settle down. At first, as in the case of the unequal pendulums, the vibration is intermittent, through a struggle, as it were, on the part of the vibrating system to assert its own period, and the frequency of the intermittence is equal to the difference of frequencies of the system and the force. In the case of an electrically-maintained fork worked by an interrupted current of a period slightly different from its own, beats are plainly heard at first. But in time these intermittent natural vibrations die away, and the system adjusts itself in some way to the time of the applied force. How this may happen will be understood from a consideration of the case of a simple pendulum, of which the point of suspension is obliged to vibrate in a given period in a horizontal line. Fig. 26 shows how the pendulum vibrates when the applied period is longer than its own. If A C B is the line in which the point of suspension vibrates, and if O C R is the length of pendulum which has the applied period; then the pendulum swings sensibly as if it were part of a pendulum of length O C R, the line of the string always passing through O. It may easily be shown, if small quantities are neglected, that this motion will give the horizontal force requisite for the vibration in the applied period.

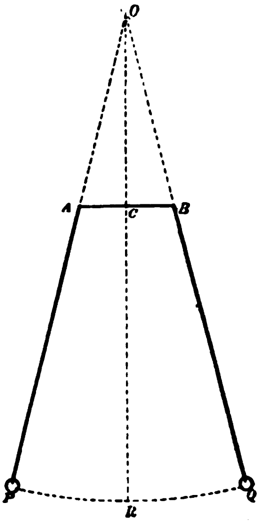


FIG. 26.—Forced Vibration of a Pendulum when the point of suspension is moved to and fro in A B in a period corresponding to length, OR, greater than its own.

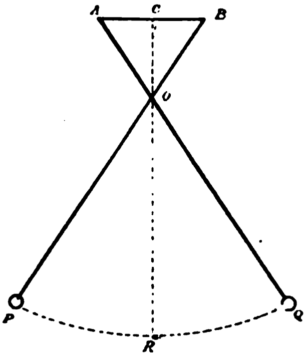


FIG. 27.—Forced Vibration of the same Pendulum when the period corresponds to length, OR, less than its own.

If the applied period is less than the natural one, the pendulum vibrates, as shown in Fig. 27, the string always passing through O, where O R is the length which would have the applied period. The pendulum then arranges its arc of vibration so that, with the given motion of the point of support, it virtually forms part of, or makes part of itself into, a pendulum of the given period. It is noteworthy that the phases of vibration in the two cases are opposed to each other.

We shall now consider a rather more general case. Let a mass M be attached to a spring, or in some way controlled, so that at distance d from its position of equilibrium the force acting on it is $-n^2 \mathrm{M} d$; the acceleration is then $-n^2 d$, and the natural period $2\pi/n$. Now let a force P sin pt, *i.e.* one having a periodic variation in time $2\pi/p$, be applied to M. Evidently M arranges its mode and extent of vibration so that the natural and applied forces together give a force of the magnitude required to make M vibrate with a certain amplitude in the applied time. The acceleration at distance d must therefore be $-p^2 d$, and the force applied and the natural force of the system must combine to give this acceleration. This requires

$$-n^2 \mathrm{M} d + \mathrm{P} \sin pt = -p^2 \mathrm{M} d$$

whence

$$d = \frac{\mathrm{P}}{\mathrm{M}} \cdot \frac{\sin pt}{-p^2 + n^2}.$$

If p is greater than n, d is positive, or the applied and natural forces agree in phase. If, however, p is less than n, they are opposite in phase. This is the result we have already seen to hold in the case of the simple pendulum with a vibrating point of suspension. It may be verified by suspending three very nearly equal pendulums, A, B, C, in descending order of length, from a flexible support. On setting B vibrating, A and C will ultimately settle down to vibrate in opposite ways.

From the resulting value of d, which we have just found, it appears that the nearer p and n are, the greater is the amplitude of vibration, and that if they are equal the amplitude is infinite. In practice there is radiation and dissipation of energy, and the amplitude only increases to the point at which the energy radiated and dissipated is equal to that put in by the working of the applied force. As the radiation and dissipation increase with the amplitude, this point is reached sometimes before the vibrations become very large, even when $p = n$.

Another result of the loss of energy is that the phase of the forced vibration is not exactly either in agreement or in opposition, but differs from that of the applied force by an amount depending on the difference of periods.

It is obvious that if we have $p = n$, it is simply a case of resonance, which may be described as forced vibration with equality of period.

CHAPTER V.

ANALYSIS OF VIBRATIONS.

CONTENTS.—Mathematical Analysis of Vibrations by Fourier's Theorem—Experimental Verification by Resonators—Quality dependent on the Constituents—Methods of Recording Vibrations—Composition of Vibrations—Vibration Microscope.

ANY vibration of a point in a line may be represented in a plane figure by a time-distance diagram, *i.e.* a diagram in which the abscissæ are the times from some chosen instant, and the ordinates the distances at those times from some chosen position. Thus a tuning-fork writing on a revolving drum traces its own time-distance diagram if the drum turns uniformly. If the vibrations exactly repeat themselves after a certain time, the curve representing them exactly repeats itself after a distance represented by the time. Such a curve is termed a **Periodic Curve.** We may conveniently term the shortest distance in which the curve repeats itself the **period** or **the wave-length**, denoting it by λ. If a line be drawn parallel to the base line, and at such a height that the curve encloses with it equal areas above and below, this line is termed **the axis of the curve.** The axis is, in fact, at the mean level of the curve. The points at which the curve cuts the axis are the **nodes.**

The curve of sines or the harmonic curve is the most important periodic curve and the simplest, in that all periodic curves may be built up out of a number of such curves as constituents. The general form of the curve of sines may be obtained by the reader if he will mark as abscissæ on a base line, say every 10°, from 0° to 360°, and then plot, as ordinates, to any scale the values of the sine as given in any table of natural sines. In this way Fig. 28 has been drawn, one curve being on half the scale of the other.

The curve evidently represents the equation—

$$y = a \sin nx \qquad (1)$$

where a is the maximum ordinate and n determines the length which shall represent 360°. We may put the equation in a more general form, which allows us to measure x from any point, not necessarily a node, and which at once gives the length representing 360°, viz.—

$$y = a \sin \frac{2\pi}{\lambda}(x - a) \qquad (2)$$

Here a is the maximum ordinate, which is termed the **amplitude.** Every addition of λ to the value of x gives the same value of y again, and this is the shortest distance in which the curve repeats itself. It is termed the **period, or the wave-length.** The successive points of intersection of the curve with the axis are alternately **ascending** and **descending nodes**, and are at intervals of $\frac{\lambda}{2}$ along the axis. The position of the first ascending node is given by a or $\frac{2\pi a}{\lambda}$, acccording as we measure by time or angle; either of these is termed **the epoch.** The angle $\frac{2\pi}{\lambda}(x-a)$ gives the position of the point on the curve, *i.e.* tells at what part of the curve it is situated, and is termed the **phase.**

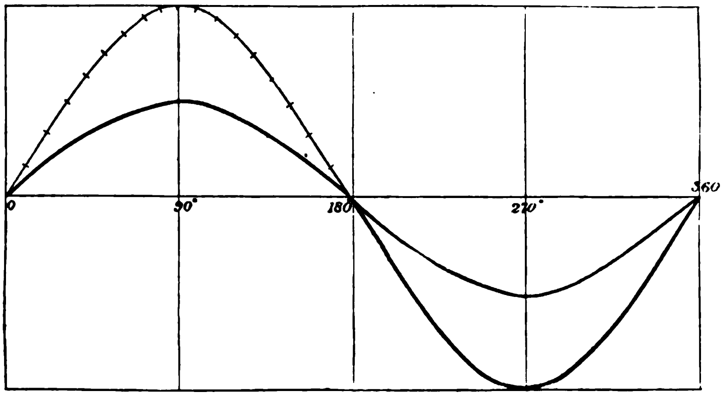


FIG. 28.—Curves of Sines. Ordinates of lower half those of upper.

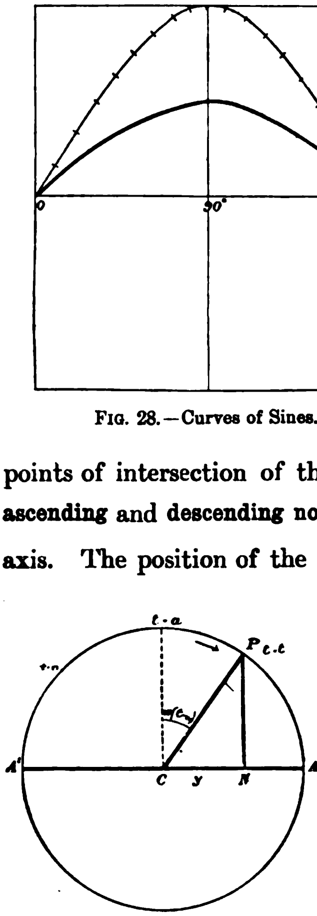


FIG. 29.—Simple Harmonic Motion. $y=a \sin w\,(t-\alpha)$.

If we take the abscissa to represent time and the ordinate distance, the curve given by equation (2) represents the time-distance diagram of a certain simple harmonic motion. For if a point P, Fig. 29, moves with uniform angular velocity, ω, round a circle of radius a, the point N, the foot of the perpendicular on any chosen diameter, AA′, possesses, by definition, a simple harmonic motion. The displacement of N is evidently

$$\begin{aligned} \mathrm{CN} = y &= a \sin \omega\,(t-\alpha) \\ &= a \sin \frac{2\pi}{\lambda}(x-a) \end{aligned}$$

if we write x for t and $2\pi/\lambda$ for ω. The periodic time of the motion is $2\pi/\omega$ or λ, the amplitude is a, and the acceleration at distance 1 is ω^2 or $4\pi^2/\lambda^2$. This may be realised experimentally by allowing a tuning-fork (which has a nearly perfect simple harmonic motion) to write its record with a nearly rigid style transversely on a uniformly revolving drum. The curve is sensibly a curve of sines, and is evidently a time-distance diagram. The curve of sines is also a curve of cosines if we measure x from a point $\frac{\lambda}{4}$ to the right of the original point; for, putting

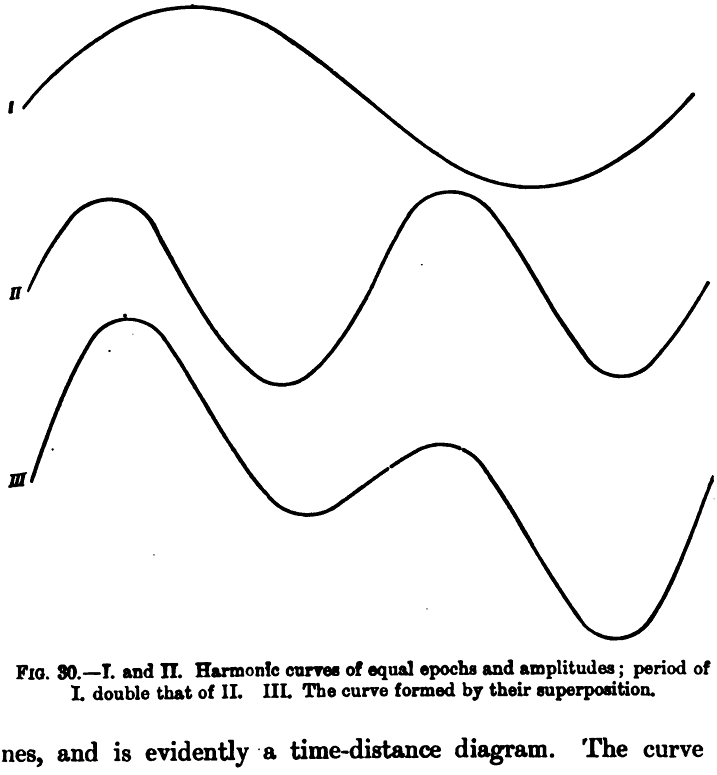


FIG. 30.—I. and II. Harmonic curves of equal epochs and amplitudes; period of I. double that of II. III. The curve formed by their superposition.

$$x = x' + \frac{\lambda}{4}$$

we have

$$\begin{aligned} y &= a \sin \frac{2\pi}{\lambda}\left(x' + \frac{\lambda}{4} - \alpha\right) \\ &= a \sin \left\{ \frac{2\pi}{\lambda}(x' - \alpha) + \frac{\pi}{2} \right\} \\ &= a \cos \frac{2\pi}{\lambda}(x' - \alpha). \end{aligned}$$

The reader will easily see that any one curve of sines may be made into any other by two simple strains, one along the axis, making the wave-length of the first equal to that of the second, and the other, perpendicular to the axis, making the amplitude equal.

Superposition of Harmonic Curves.—If a curve be drawn with the ordinate at any point equal to the algebraic sum of the ordinates of two or more curves at the same point, or at points with equal abscissæ, it is said to be formed by their superposition. In Fig. 30, III. is formed by the superposition of I. and II., and the reader may verify this by measuring the ordinates of I. and II. for equal abscissæ. Their algebraic sum will be found equal to the ordinate of III. for the same abscissa.

There is one case of superposition worthy of special notice, that in which the two components are harmonic curves which have the same period. The reader should draw two such curves, and show graphically that the result of their superposition is a third harmonic curve of the same period. Analytically, this can be shown from the equation—

$$y = a \sin \frac{2\pi}{\lambda}(x - \alpha) + b \sin \frac{2\pi}{\lambda}(x - \beta)$$

which can be reduced to

$$y = c \sin \frac{2\pi}{\lambda}(x - \gamma)$$

where

$$c^2 = a^2 + b^2 + 2ab \cos \frac{2\pi}{\lambda}(\alpha - \beta)$$

and

$$\tan \gamma = \frac{a \sin \frac{2\pi}{\lambda}\alpha + b \sin \frac{2\pi}{\lambda}\beta}{a \cos \frac{2\pi}{\lambda}\alpha + b \cos \frac{2\pi}{\lambda}\beta}.$$

Superposition of Harmonics with Commensurable Periods.—A periodic curve may be regarded as repeating itself, not only after one period or wave-length, but also after two, after three, or after any whole number of periods. If, then, any number of harmonic curves have periods each some aliquot part of a given period, each will repeat itself after this period; and if they be superposed, their sum will also repeat itself, or will form a periodic curve of the given period. Not only is it true that harmonics of commensurable periods will build up into a periodic curve, but it can also be proved that, conversely, any periodic curve whatever, if only of such kind that it is a possible time-distance diagram, may be built up of harmonics of periods which are aliquot parts of the period of the curve. The general statement of this most important result is as follows, and is known from its discoverer as

Fourier's Theorem.—Any periodic curve of period λ, which is continuous, always at a finite distance from the axis, and with

wave length

only one ordinate for each abscissa, may be formed by the superposition of harmonic curves of wave-lengths $\lambda, \frac{\lambda}{2}, \frac{\lambda}{3}, \frac{\lambda}{4}$, by properly adjusting the epochs and amplitudes of the constituents, and there is only one set of harmonics which will thus form it.

The reader may be referred for the proof of this theorem to mathematical treatises such as Donkin's *Acoustics*, where he will find rules for determining the epochs and amplitudes of the various constituents when the form of the periodic curve is given. For our purpose the general statement is sufficient.

One particular case should be specially noted—that in which the beginning, half-way point, and end of the periodic curve are on the axis or have zero ordinates. In this case, all the constituents have nodes at the same three points.

The harmonic curves of wave-lengths $\lambda, \frac{\lambda}{2}, \frac{\lambda}{3}$, &c., are described as the harmonic series for a periodic curve of length λ, and each is termed a **harmonic of the curve.** We shall illustrate the importance of Fourier's theorem in the theory of sound by stating here some results of the investigation on the transverse vibrations of stretched strings given fully in the next chapter.

Let us suppose a stretched string of length l, with its ends fixed, to be slightly displaced from its equilibrium position into a plane curve of any form, and then to be suddenly released. The curve of initial displacement may be regarded as one-half of a periodic curve of length $2l$, with nodes at the two ends of the string. By Fourier's theorem it may be resolved into harmonics, all with nodes at the two ends, of wave-lengths $2l, \frac{2l}{2}, \frac{2l}{3}$, and so on. Now, when a string is stretched into a curve of sines and let go, it may be shown that it will always remain a curve of sines with the same nodes and wave-length, the amplitude only changing. The time of vibration is proportional to the wave-length. Further, if a number of such displacements coexist, each may be considered to exist independently of the rest. It therefore follows that all the harmonics in the original displacement will persist, and will have times of vibration, T, T/2, T/3, . . . where T is the time of the harmonic of length $2l$. Evidently, after time T the string will return to its initial position, and not before that time. Since all the harmonics persist, and since there is only one way of building up a curve by harmonics, no new harmonic can come in in the course of the vibration.

We may regard each mode of vibration as sending out its own wave into the surrounding air, or through the supports of the string. Or, taking the complete waves sent out by the complete vibrations of the string, we may analyse these waves into their harmonics by Fourier's theorem, and we get the same result.

The Existence of these Harmonic Vibrations may be detected by Resonance.—Let a second string be exactly tuned to unison with any one of the harmonics of the original string, and let it be in material connection with it—say on the same resonance-box. Then, if that harmonic is present in the vibration of the first string, the second string will take up energy from it, and will be set in vibration. It must be remembered that not every harmonic will be present, for the amplitude of some in the original displacement may be zero, so that the second string will not always vibrate, even when its time of vibration is an aliquot part of that of the first. It is always easy to detect at least two harmonics by the foregoing method in a piano. Holding down the key of *c* and striking C several times sharply, *c* will be heard sounding when C is silent again. Or holding down *g*, the next harmonic of C, it too may be made to sound; and c' may probably be obtained.

The harmonics may also be detected by "Resonators," or hollow globes with orifices, tuned so that the air within vibrates with a particular tone. The usual form of resonator, which is due to v. Helmholtz, is shown in Fig. 31, a small tube which is applied to the ear allowing the vibrations to be plainly heard. With a series of resonators forming the harmonic series for a given note, it is easy to determine which harmonics are present when that note is sounded on a piano or organ.

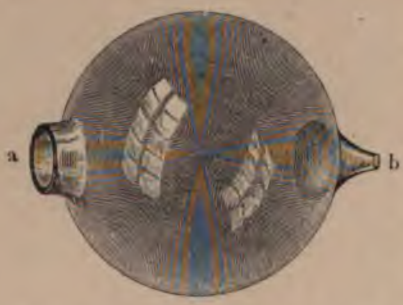


FIG. 31.—Helmholtz's Resonator. Usually of thin brass. A wide orifice, *a*, at one side to receive the exciting waves; a narrow orifice, *b*, at the other to conduct the vibrations to the ear.

Simple Tones.—If a vibration is a simple harmonic motion of small extent, it gives rise to waves of which the displacement diagrams are harmonic curves or curves of sines. These are, as far as Fourier's theorem is concerned, simple curves, since they cannot be further resolved thereby. The sounds which they excite are termed **Simple Tones.**

The Ear probably Contains a Series of Resonators.—It is very probable that the ear is a kind of practical Fourier's theorem. It is supposed that certain fibres—the fibres of Corti—are tuned so as to respond each to a particular vibration, or at least to vibrations within a narrow range. When a series of compound waves disturbs the auditory apparatus, each fibre takes up its own waves and so breaks up the compound into its constituents. But this is perhaps rather a speculation than, as yet, an ascertained fact.

An account of the subject will be found in v. Helmholtz's *Sensations of Tone*, ed. 1875, chap. vi. There can be no doubt, however, that in a mixture of tones, as in a harmony with the various parts, each tone excites a separate sensation; for, if the attention is properly educated, it is easy to follow the different parts. Even when a single note is played, an ear trained for the purpose will be able to detect some of the harmonics.

QUALITY OF MUSICAL SOUND.

In the first chapter we stated that the quality of a musical sound depends on the form of the waves exciting it. Taking the conventional representation of the waves by a displacement diagram, we have for a series of regular waves a periodic curve, and we may say that the quality depends on the form of this curve.

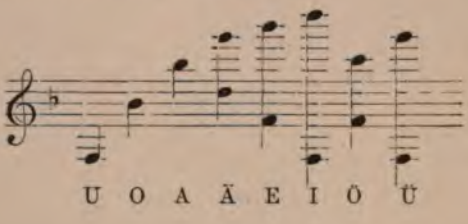
Applying Fourier's theorem, each wave may be broken up into a number of harmonics, and the number, amplitude, and epoch of these will vary with the form. For any given form there will be a perfectly definite set of constituents definitely arranged, which may be regarded as building up that form. The quality of a sound which corresponds to regular waves depends, therefore, on the harmonics into which the waves can be resolved. It is convenient to use the term **note** for an ordinary compound sound to which a definite pitch may be assigned, and the term **tone** for each simple harmonic constituent which goes to form it.

A very important question arises now, viz., does the quality depend on the relative phases or on the epochs of the constituents of a note? Some apparently quite decisive experiments were made by v. Helmholtz which gave a negative answer to this question. He used as a source of sound a cylindrical hollow metal resonator, with a circular hole in one end, which could be closed to any desired extent by a sliding circular lid. This was placed with the hole close to an electrically maintained fork with which it was in unison, and consequently there was resonance. When the hole was quite open the unison was perfect, and there was complete agreement in phase. But on gradually closing the hole by the lid, the natural frequency of the resonator was raised more and more above that of the fork, and theory shows that the forced vibrations of the resonator differed more and more in phase from the exciting vibrations of the fork with which they were forced to agree in frequency. A set of forks with resonators of this kind was prepared, consisting of B♭ and the next seven harmonics. When any combination of these forks was made to sound, and the phases were varied while the intensities were kept practically constant, the variations in phase could not be observed to produce any change in quality; while the quality was quite altered by changes in the number and strength of the constituents.

An account of the mixture of harmonics giving their peculiar quality to the notes of various common musical instruments will be found in Helmholtz's *Sensations of Tone*, chap. v.

The Components of Vibration are sometimes Inharmonic.—In many cases, especially when the restoring force of the vibration is due to the rigidity of the material, the frequencies of vibration of the constituents of a note are not harmonics. In a tuning-fork, for example, the frequencies of the simple tones making up the note given out are in ratios 1 : 6·25 : 17·5 : 34·3, &c. In such a case Fourier's theorem does not give us the forms of the different modes of vibration. Thus, suppose that we have a fork vibrating only in one period, and giving its fundamental note alone, its form will not be a curve of sines or any part thereof. Conversely, if we displace the fork so as to make it take the form of a part of a curve of sines, on being released it will not maintain that shape, but will break up into constituents of different shape. In fact, Fourier's theorem is only one particular mode of resolving a curve, which is the easiest in its mathematical investigation, and which happens to correspond to the physical properties of certain vibrating instruments. But in other cases other modes of resolution are applicable, and other series of tones are then obtained. In these cases the higher tones obtained are termed **overtones** or **upper partials.**

Vowels.—The most interesting example of the correspondence of quality of sound to a given mixture of tones is given by the various vowel sounds produced by the human voice. The organ of the voice is essentially a double reed supplied with resonance cavities in the throat, mouth, and nose, the shape of which the speaker can vary at will. Each vowel sound has in it certain definite tones, whatever the fundamental tone in which it is uttered, the resonance cavities above the vocal chords being adapted to reinforce these particular overtones in the sound uttered by the vocal chords. This was first stated by Wheatstone. Helmholtz, to whom we owe the most extended investigations on the subject, produced the vowel sounds by various mixtures of simple tones produced by resonators to tuning-forks, though before him various investigators, among whom Willis had been most successful, had succeeded in producing the vowels artificially. Helmholtz gives the following as the tones forming the basis of the vowels written under them :—



Different investigators differ in their determinations of the vowel constituents, and this we might expect, since common experience sufficiently shows that no two voices agree in giving the same quality to a given written sound.

GRAPHIC MODE OF RECORDING VIBRATIONS.

The Phonautograph.—This is an instrument for recording the vibrations corresponding to various sounds, writing them down entire without any analysis. It consists of a parabolic reflector, with a membrane stretched across the narrow end at the focus. At the centre of the membrane, on the side away from the reflector, is a style which writes on a revolving drum covered with smoked paper, the motion of the style being parallel to the axis of the drum. The axle of the drum being cut with a screw-thread, the drum moves forward as it is turned round. Any series of waves sent into the cavity of the reflector set the membrane vibrating, and its motion is recorded on the drum. This instrument is chiefly of interest now in that it was a great step towards a most remarkable invention, viz.—

Edison's Phonograph.—This is an instrument which not only records the sounds which it receives, but also reproduces them.

Like the phonautograph, it consists of a stretched membrane, with a style at its centre recording on a drum; but instead of making the vibrations parallel to the axis, the style moves in and out, ploughing its way through the surface of the material of the drum, and recording the vibrations by a furrow of varying depth. In the original form of the instrument, Fig. 32, the drum had a screw-thread on it of the same pitch as that on the axle, and a sheet of tinfoil was wrapped round the drum. The style, which had a blunt point, indented the foil along the screw-thread, and the varying depth of the indentation recorded the vibrations. The drum was brought back to its original position and the style was allowed to press against it, so as always to be at the bottom of the indentation. On turning the drum at the same rate as before, the membrane was made to execute the vibrations recorded on the foil, and so gave out again the waves at first put in. But the membrane did not record all the original vibrations in the same proportion, so that the reproduced sounds were very different in quality from the original ones, though words could often be easily recognised. In later forms of the instrument devised by Tainter and Bell, as well as by Edison, the tinfoil cylinder is replaced by one of wax, say 4 inches long and $1\frac{1}{2}$ inches in diameter, which can be turned by a motor at a very exactly governed speed. A cutting-tool first turns a smooth screw-groove in the wax, and then the cylinder is brought back to the initial position. A mouthpiece closed by a membrane, which may be of glass, say $\frac{1}{200}$ inch thick, is placed in position so

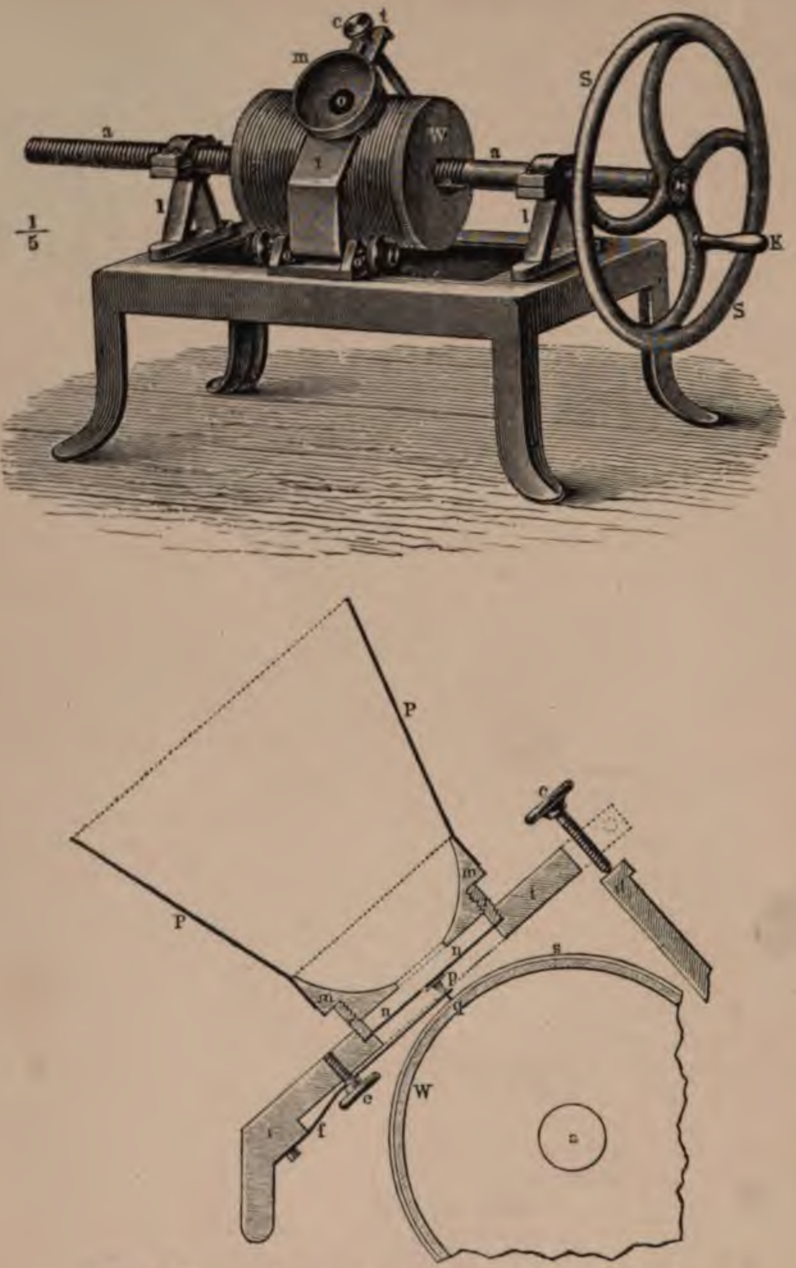


FIG. 32.—Phonograph. a a, axis with screw-thread turned in bearings ll by the handle K; W, cylinder with screw-thread of same pitch as a a, and covered with tinfoil; m, mouthpiece, of which a side view is given in the lower figure; n, membrane connected with style p.

that a recording style, which is fixed to the centre of the membrane, may reach to the bottom of the groove. The cylinder is then set in motion, and the words to be recorded are spoken into the mouthpiece, and duly imprinted by a series of varying depths in the wax. To reproduce the sounds, a second membrane, say of oiled silk, with an attached blunt point in place of a style, is brought into the position formerly occupied by the recording membrane, the point being pressed slightly against the bottom of the groove in the wax. When the cylinder is set in motion the original sounds are reproduced, and, under favourable circumstances, the original quality is exceedingly well preserved.

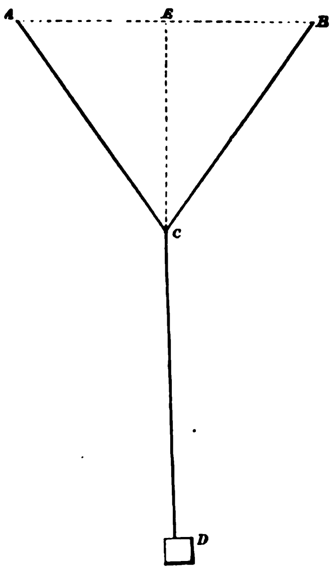


FIG. 33.—Pendulum hung by string so as to execute two vibrations at right angles, one in the plane of the figure corresponding to length CD, the other in a perpendicular plane corresponding to length ED. The times are in the ratio $\sqrt{CD/ED}$.

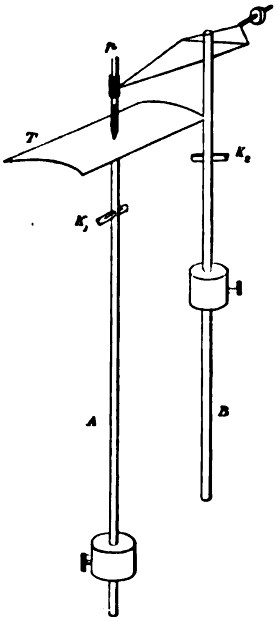


FIG. 34.—Diagram of Harmonograph. The pendulum A vibrates on the knife-edge K_1 (support not shown), giving a harmonic motion to a cylindrical table T at its upper end, K_1 being the axis of the cylinder; B vibrates on the knife-edge K_2, which is at right angles to K_1, and gives its motion through a jointed framework to the siphon pen *p*. This moves harmonically in a line parallel to K_1, and therefore, relative to the table, has a motion compounded of the two motions. The paper is attached to the surface of T.

The Composition of Vibrations at Right Angles.—This subject may be usefully entered into at this point, though it has no direct connection with the foregoing discussion of the analysis of vibrations. We shall see, however, that it leads to a method of detecting the form of vibrations which is of considerable importance—that of the Vibration Microscope.

A simple mode of exhibiting the composition of two vibrations at right angles is shown in Fig. 33. A string, ACB, is fastened to two points, AB—say two nails in a doorway—and to its middle point C a third string CD is attached, having a heavy weight, D, at its lower end. If the weight is drawn aside in the vertical plane through AB, it vibrates as a pendulum of length, CD, and if in the vertical plane through ECD, it vibrates as a pendulum of length, ED. If drawn aside in any other way, and either released or projected, its motion is a resultant of these two motions. The motion may be recorded by making the weight D hollow, with a small hole at the bottom, and filling it with sand. The sand trickles out, and marks, on a sheet placed below, the curve traced out. By varying the ratio ED/CD and the relative phases and the relative amplitudes, the figures obtained may be varied to any extent.

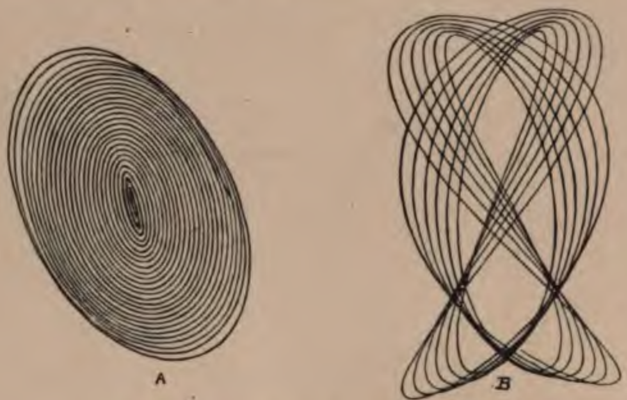


Fig. 35.—Curves drawn by a Harmonograph.
A. Imperfect unison. *B.* Imperfect fifth.

The Harmonograph.—The simple apparatus just described has been developed into the harmonograph, which consists of two pendulums vibrating at right angles and communicating their motion to a glass siphon pen, which records on paper the resultant motion. Fig. 34 gives a diagram of one arrangement of the instrument, where the pen has one motion and the paper the other. Owing to the gradual decrease of the vibrations and the consequent interlacing of the successive curves traced out, it is most interesting to watch the gradual development of the figures. These are often of great beauty. Fig. 35 represents an imperfect unison and an imperfect fifth.

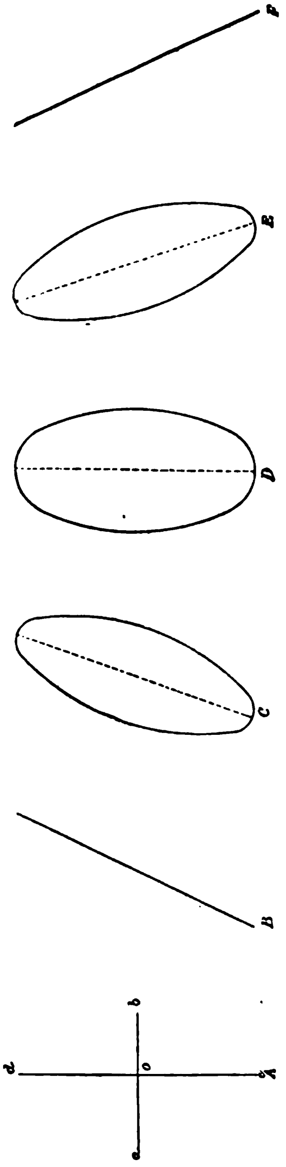


FIG. 36.—A, Two directions of vibration; frequency in *a b* supposed greater than that in *c d*. B, Resultant when the two start in the same phase along *o a* and *oc*. C, D, E, F, successive forms as the phase of *c d* lags behind that of *a b*.

The Kaleidophone.—If a knitting-needle is clamped in a vice and a small bead is fixed on its free end, the image in the bead of any bright source of light will appear to describe a continuous curve when the needle is set vibrating. If the needle is quite round, its time of vibration in any direction will be the same, and the curve will be either a straight line, an ellipse, or a circle. But if there is some want of uniformity, there will be two directions at right angles to each other of greatest and least time of vibration, and the curve will be the resultant of vibrations in these two directions. If the two frequencies are nearly the same, and the vibrations begin in the same phase, the curve will be a straight line, as in Fig. 36, B, slowly changing into an ellipse, C, of which the axis revolves, the ellipse at the same time becoming more circular till one vibration has gained $\frac{1}{4}$ period on the other as at D; the ellipse then contracts again through the form E to a straight line as at F, when the gain is $\frac{1}{2}$ a period, and so on.

This mode of exhibiting vibrations is developed in Wheatstone's kaleidophone, which consists of a number of rods of various rectangular sections arranged so as to vibrate with frequencies in various ratios along the two axes of the rectangle. The vibration is shown by an illuminated bead. Fig. 37 shows the resultants of several exactly timed intervals with various differences of phase. It will be observed that on circumscribing a right angle round any figure with its sides

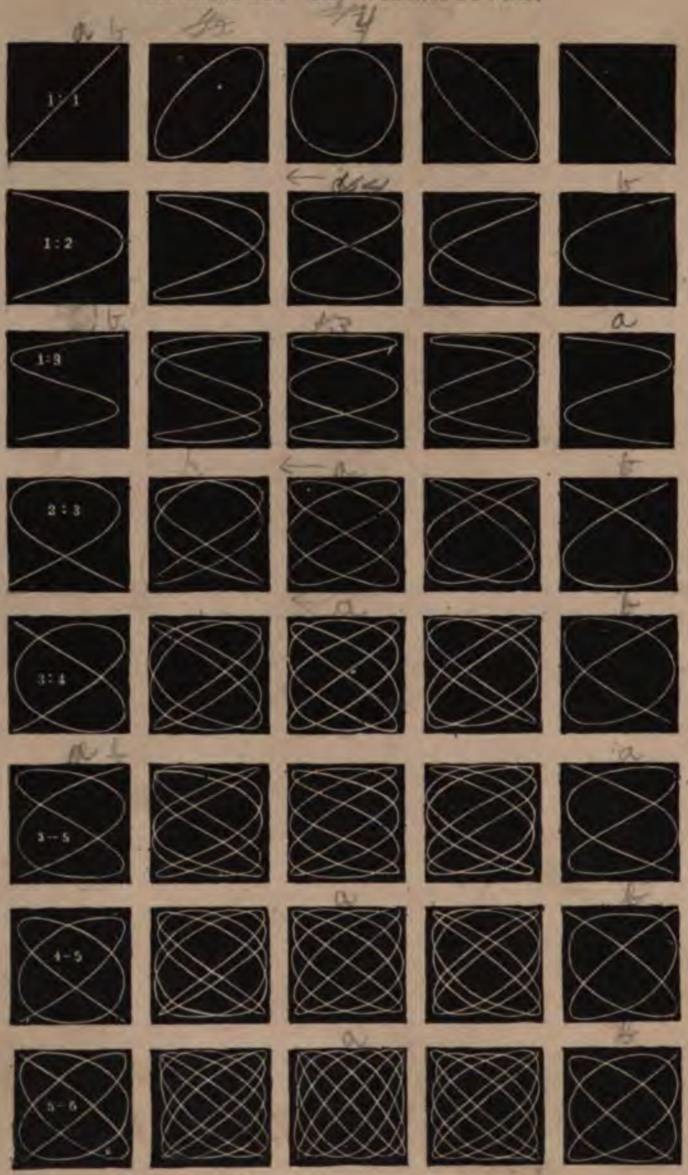


FIG. 37.—Kaleidophone figures, various intervals with various differences of phase.

horizontal and vertical, the ratio of the number of contacts with adjacent sides gives the ratio of the frequencies.

Beautiful figures may be obtained by bringing out upper partials with the fundamental notes of the rods.

A single compound rod, the Universal Kaleidophone of Lippich and Melde, may be made to give all the figures. This consists of a narrow vertical brass strip fixed on a base, and carrying at its upper end a brass piece with a slot in it. In the slot can be fixed a second vertical strip in continuation of the first, but not quite in the same vertical line. The plane of the second strip is at right angles to that of the first, and the length of it above the slot is adjustable. At its upper end is the illuminated bead.

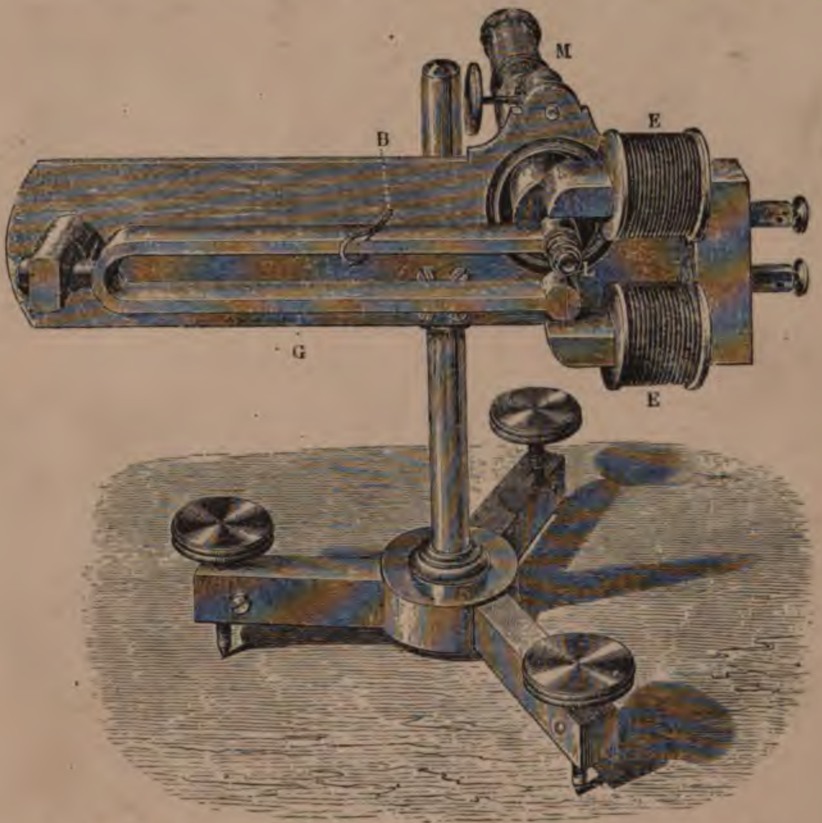


FIG. 38.—The Vibration Microscope. L, objective of microscope attached to one prong of fork; M, fixed eye-piece; E, maintaining electro-magnet. The vibration to be studied is horizontal.

Lissajous' Figures.—Lissajous obtained similar figures by compounding the vibrations of two tuning-forks moving in perpendicular planes. If small mirrors are placed on one prong of each fork and a ray from some small source is received in a telescope after it has been reflected successively at the two mirrors, the image of the source describes Lissajous' figures. The method admits also of projection on a screen by suitable arrangement.

Vibration Microscope.—This instrument may be regarded as a development of Lissajous' method, and was used by Helmholtz, not so much to determine frequencies as the actual form of vibration of particular points in strings. It is represented in Fig. 38, and consists of a tuning-fork which carries at the end of one prong the objective, L, of a microscope with fixed eye-piece, M. The fork is electrically maintained by the electro-magnet E. B is a small weight for adjusting the pitch to a slight extent. The string, of which the vibration is to be investigated, is fixed vertically in the field of view of the microscope with a starch grain on it to serve as a luminous point. If the fork alone moves, the point is lengthened out into a bright vertical line, but when the string also moves the point traces out a curve. In Fig. 39 *cd* represents the curve given in a particular case by a violin-string in unison with the fork. *ab* is the vibration due to the fork alone, and since this vibration is harmonic, we at once get a time scale by drawing a

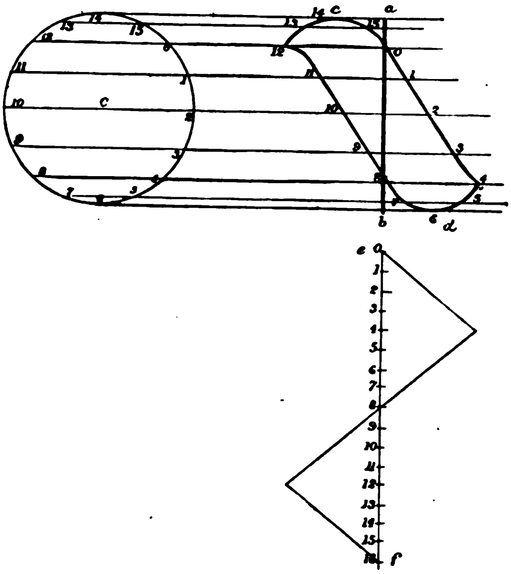


FIG. 39.—Curve of violin-string given by vibration microscope with which it is in unison; *ab*, line seen when fork alone vibrates; *cd*, curve seen when the string also vibrates. C, time circle with diameter equal to *ab*. The dotted lines being drawn through equidistant points on the circumference of C, mark off equal times. *ef* is the time displacement diagram for the vibration of the string, the time scale being vertical.

circle with diameter *ab*, and drawing a series of horizontal lines through equidistant points on the circumference. Where these meet the curve *cd*, we have the positions of the point at successive intervals of tune.

Plotting out the horizontal ordinates at the successive instants on a new time scale, *ef*, we get the time displacement diagram of the vibration of the point on the string.

CHAPTER VI.

THE TRANSVERSE VIBRATIONS OF STRETCHED STRINGS OR WIRES.

CONTENTS.—Experiments with the Sonometer—Harmonics: Their Coexistence—Mechanical Investigation of Transverse Vibrations—Propagation of Transverse Waves along Strings—Stationary Waves—Modes of Vibration deduced from Stationary Waves.

IN Sound a "string" signifies a cord or wire stretched between two fixed points, with a tension so considerable that if the string is slightly displaced and released it vibrates to and fro, and gives out a musical note. In general the applied tension is the only restoring force which we need to consider, for with small displacements and thin wires the flexure is usually negligible, and the forces due to extra extension on displacement are exceedingly small compared with the initial tensions applied in practice.

The various "stringed" instruments, as the piano, the violin, the harp, afford examples of the kind of strings of which we are now to investigate the modes of vibration.

Experimental Investigation by the Sonometer.—The Sonometer (Fig. 40) consists of a piano wire or violin string, *ab*, stretched across two bridges on the top of a hollow wooden "sounding-box." It is fixed at one end and passes over a pulley at the other end, so that it can be loaded to have any desired tension. It is convenient to have a second comparison string, *cd*, which can be tuned to any pitch by means of a wrest-pin. There is a movable bridge, *f*, sliding over a scale of lengths, *ss*. This can be placed so as to fix the string *ab* at any point; or, turning it round, it may be used to fix any point of *cd*. With such an arrangement we may at once verify the following laws.

The vibrations are isochronous for a given string with given tension.—If the string is plucked, the same note is heard all the time as the sound dies away, though the amplitude is continually decreasing. In other words, the pitch is independent of the extent of swing.

The frequency of vibration is inversely as the length with given tension.—Tuning the two strings to unison, place the bridge *f* half-way along *ab*, when it will be found to give the octave of *cd*, vibrating therefore with twice the frequency. Place *f* one-third of the way along *ab*, and one part will give the fifth and the other the twelfth, having frequencies respectively $\frac{3}{2}$ and 3 times that of *cd;* and so on, the frequency always being inversely as the length.

With constant length the frequency is proportional to the square root of the tension or stretching weight. W.—Starting with a given load and the strings in unison, put on four times the load, and *ab* will now give the octave of *cd*. Put on nine times the load, and *ab* will give the twelfth of *cd*. Subdividing *cd* by the bridge, we may easily verify the accuracy of these statements, and easily, also, verify the correctness of the law for any other loads.

The tension and length being the same for two strings, their frequencies are inversely as the square roots of the mass per unit length, and are independent otherwise of the material.—This may be verified by stretching in succession two strings of known mass per unit length with the same load, and determining the ratio of the frequencies by the comparison string of the sonometer. If the strings are of the same material, their masses per unit length will be as the squares of their diameters, and their frequencies inversely as their diameters. The lengths of the comparison string in unison respectively with the two will be directly as their diameters.

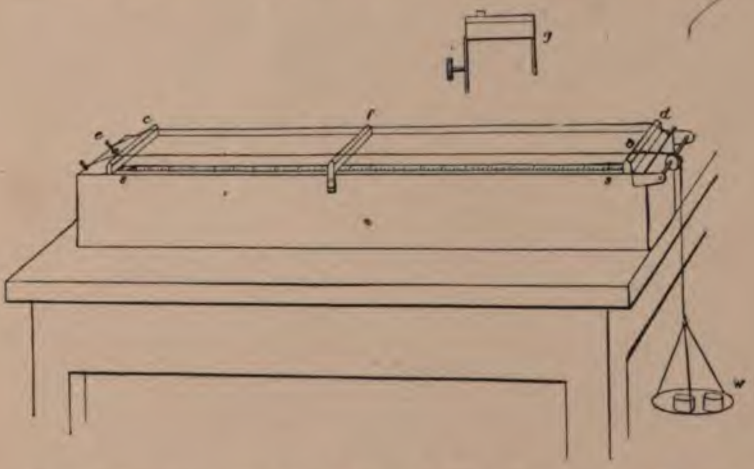


FIG. 40.—Sonometer.—*ab*, string with tension varied by the weight **W**; *cd*, comparison string with tension varied by screwing up at the wrest-pin *e*; *f*, movable bridge so arranged that either string may be damped by it without the other; *g*, movable bridge shown separately; *ss*, scale.

That the frequency should be independent of the material, except as this affects the mass of a given length of string, is just what we should expect if the restoring force is merely due to the tension to which the string is subjected.

Summing up the three foregoing results, if n be the frequency of vibration, l the length, T the tension, and m the mass per unit length—

$$n \propto \frac{1}{l}\sqrt{\frac{T}{m}}$$

The Harmonics of a String.—If a string is plucked at the centre or bowed, it vibrates as a whole, giving out its characteristic note, the fundamental note. But if the string is "damped," or lightly pressed by a quill or the edge of a paper-knife at the middle point, and either half is bowed, the octave is heard. If the string is damped a third of the way along, and the shorter length is bowed, the twelfth, with three times the frequency of the fundamental, is heard. Damping in succession $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, and so on from one end, and always bowing the shorter length, notes of 4, 5, 6, and so on times the frequency are obtained. The series of notes so obtained are termed the **harmonics of the fundamental note.** It

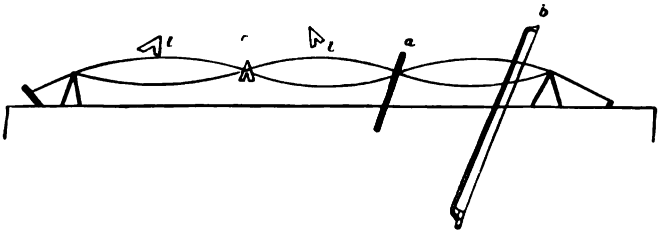


FIG. 41.—Nodes and loops of a string (greatly exaggerated) when the string is damped at a and bowed at b. The rider remains on at the node n, but those at the loops ll are dismounted.

is usual to describe the octave as the first, the twelfth as the second, and so on; but it would be more in accordance with the mathematical treatment of the subject, which makes these harmonics correspond to the successive terms in the Fourier analysis of the vibration, if we included all the tones in the harmonic series, the fundamental being the first, the octave the second, and so on; and this course we shall here adopt.

A simple experiment shows that when the string is damped at any of the points described above and bowed, it divides into a number of vibrating segments, each equal in length to the bowed segment and separated by points at rest. Cutting a number of V-shaped pieces of paper and placing these inverted, as riders, along the string, it will be found that they are thrown off if in the middle of a segment, and that they remain on if at the separating point between two segments (Fig. 41). These separating points are termed **nodes**, while the vibrating portions of the string are termed **loops**. Though the nodes are described as at rest, this only means that their motion is very small compared with that of the loops. Really, of course, they must move slightly, for the energy of vibration comes through them from the bowed segment. And the "fixed" ends of the string, which are also nodes, must have some motion, for through them the energy passes to the sounding-box or board on which the string is mounted.

Melde's Experiment.—This experiment exhibits in a most striking way the fact that a string may be divided into any number of equal vibrating segments. One end of a light silk cord is fastened to a prong of a tuning-fork, while the other passes over a small pulley and has a variable weight attached to it. The vibrations of the fork may be either along the string, as in Fig. 42, A, or transverse, as in Fig. 42, B. If the fork is bowed or electrically maintained, and the length of the cord and stretching weight are properly adjusted, the cord is set into vibration, the apparently continuous loops being separated by the sharply defined nodes. If n is the frequency of vibration of the string, a constant quantity depending only on the fork, the formula (see above, p. 82) gives $nl \propto \sqrt{\text{Tension}}$. If, then, we find W to be the weight necessary to make the cord vibrate as a whole, $\frac{W}{4}$ will make it vibrate in two segments, $\frac{W}{9}$ will make it vibrate in three segments, and so on. There is a difference between the two positions A and B of the fork. In A, when the prong is in its further position outward, the string is slack and at the extremity of its swing. When the prong is at the end of its inward swing the string is stretched tight. Hence the string makes only ½ of a vibration,

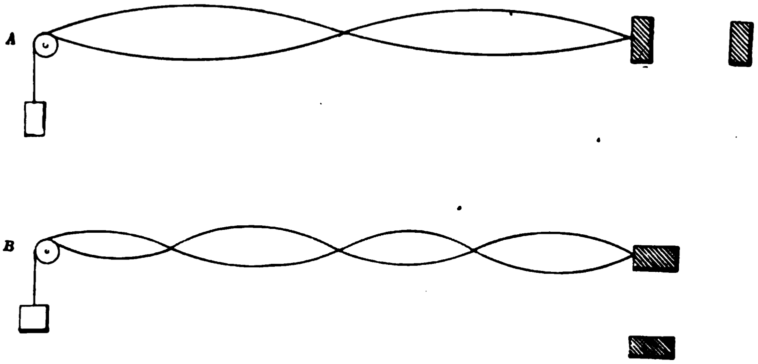


FIG. 42.—Melde's Experiment.—A, vibrations of fork longitudinal; frequency of fork twice that of string. B, vibrations of fork transverse; frequencies of fork and string equal. Twice as many loops on B as on A for given tension and length.

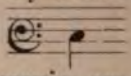
while the fork makes $\frac{1}{2}$ of a vibration. In other words, the frequency of the string is half that of the fork. In B the fork swings the string to and fro with it, and the two have the same frequency. Consequently, with a given fork and a given weight there are twice as many loops in the position B as in A, for the frequency being doubled, the length must be halved.

It will often be seen that in the position B the fork is not quite at a node, but at a point near the node with the same amplitude of vibration as the prong.

From all these experiments we conclude that—

A string may vibrate as a whole, or in any number of equal parts, its frequency being proportional to the number of parts.

The Co-existence of Harmonics in the Note of a String.—When a string vibrates, it gives out really a mixture of tones, the fundamental note by which we describe it being only the first of a series of harmonics. The particular members of the series present depend on the mode in which the vibrations are excited.

Any one with an ear capable of recognising pitch may obtain some verification of this statement from a piano. If *c*  is taken as the fundamental note, its succeeding harmonics are *c*′, *g*′, *c*″, *e*″, and so on. Let the experimenter strike these notes in succession, and fix them in the mind. Then, when all sound has died away, let him strike *c* sharply and keep the key down so as to prolong the sound. He will certainly hear some of the harmonics, especially as the sound gets faint; for the relative intensities of the harmonics change considerably with gradual cessation.

While the ear alone will, after a little training, easily recognise the presence of some of the higher harmonics accompanying the fundamental tone, they can most certainly be detected by calling in the aid of the Principle of Resonance. If the two strings of the sonometer, for example, are tuned to exact unison, and one of them is then divided by the movable bridge, so as to give one of the harmonics when vibrating on its own account, it will be set in vibration when the other string is struck, if that harmonic is present. For this experiment exact tuning is necessary. A similar experiment may be made with a piano. Let *c*′ be pressed down so gently as not to sound, and let it be held down. Let *c* now be struck sharply, and then silenced by damping. It will be found that *c*′ is sounding distinctly. In the same way *g*′ and *c*″, and perhaps higher harmonics, may be obtained, though the equal temperament tuning throws the strings too far out of the harmonic series in some cases. In other cases, they are just far enough out for beating to occur between their natural vibrations and the forced vibrations of the harmonic of the note sounded.

Helmholtz's resonators, or their rough equivalents, narrownecked vases, or narrow tin canisters, may also be used to detect particular harmonics.

The co-existence of the harmonics in a note explains a result which may easily be obtained with the sonometer. If the string be bowed and then damped lightly at one of the nodes for any harmonic, that harmonic will, if initially present, continue to sound. Thus, if the string is damped at the middle point, the octave sounds. If at a third of the way along, the twelfth sounds. But these must have been initially present, though masked by the fundamental. If the string is bowed *very exactly* at the node of a given harmonic, that harmonic is absent; and if the string is damped at the point of bowing, the harmonic is not heard. Indeed, if the bowing and damping are exactly at the same point, the string ceases to sound altogether. For the bowing only introduced harmonics which had no node at the point, and all these the damping extinguished.

Note, Tone, Fundamental Tone, Overtone, &c.—It may be useful to recall here the definitions of these various terms already given. When a source, such as a string, sends out a musical sound, the whole sound is termed a **note**.

Any musical ear assigns a definite pitch to this note, and its frequency may be determined. But in reality the note is a compound of notes, and the pitch assigned is only that of the lowest of the series. Each of the components, itself irresolvable, is termed a **tone**, the lowest the **fundamental tone**, the rest **overtones** or **upper partial tones**. In the particular case in which the overtones have frequencies exact multiples of that of the fundamental, the whole series are **harmonics**.

MECHANICAL INVESTIGATIONS OF THE MODES OF TRANSVERSE VIBRATION OF A STRING.

First Method: Vibration of a string displaced to form a Curve of Sines.—In investigating the motion of a string disturbed in any way and then left to itself, it is usual to form a differential equation to represent the motion and to obtain the most general solution of this equation.

But if we assume the truth of Fourier's theorem, it is possible to build up an equally general solution of the problem involving only elementary mathematics, and as this method appears to bring out clearly the appropriateness of Fourier's theorem to the problem, it is the plan which we shall follow here.

Applying the theorem to a string stretched between two points, we may consider any curve into which it is displaced as one-half of a periodic curve, with nodes at the beginning, middle, and end of the axis. Hence it can be resolved into a series of harmonic curves, with one, two, three, &c., arches or loops, all beginning and ending—*i.e.*, all with nodes—at the fixed points. Considering one of these harmonics alone, we shall first investigate

The motion of a string displaced to form one arch of a curve of sines and then released.

Let APQB (Fig. 43) represent such a curve formed by a string of length $l=\lambda/2$, mass m per unit length, and stretched with tension T, the equation to the curve being

$$y=a \sin 2\pi x/\lambda.$$

Let PQ represent an element of the string which is acted on by the two end tensions, T at P along the tangent RP, and T at Q along the tangent RQ. Let PR meet the ordinate at Q in T. If we replace PQ by the same arc of the circle of curvature drawn on a reduced scale in (2), the sides $\mathrm{OP}=\mathrm{OQ}=\rho$ of the triangle OPQ may represent the two equal tensions at P and Q, since they are equal to each other and perpendicular to the forces.

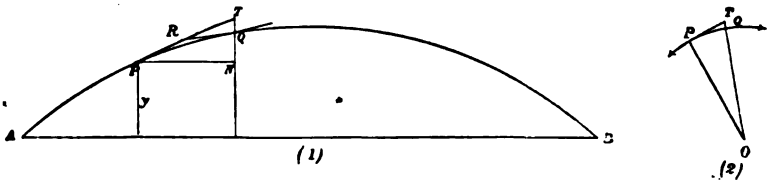


FIG. 43.—(1) APQB, one arch of a curve of sines of length $l=\lambda/2$. The vertical scale greatly magnified. PQ an element also magnified, PT the tangent at P meeting the ordinate through Q in T. At the right hand (2) is drawn the circle of curvature at PQ. The curvature is here greatly magnified, so that $\mathrm{OP}=\rho$ is much diminished. In the actual case both PO and QO are so nearly perpendicular to the axis AB, that T may be taken as representing the same point in both figures.

On the same scale, then, PQ will represent the resultant, and be perpendicular to it. But as the string is really only very slightly displaced, PQ is sensibly parallel to the axis, and the resultant is therefore perpendicular to the axis. Its magnitude is

$$\mathrm{T}.\frac{\mathrm{PQ}}{\mathrm{OP}}=\frac{\mathrm{T}}{\rho}\,\mathrm{PQ}.$$

Since the mass of PQ is m. PQ

$$\text{acceleration of PQ}=\frac{1}{\rho}\,\frac{\mathrm{T}}{m}\cdot$$

From Fig. 43 (2) we have approximately—

$$\mathrm{TP}^2=\mathrm{PQ}^2=\mathrm{TQ}(\mathrm{TQ}+2\rho)=2\mathrm{TQ}.\rho$$

$$\therefore \frac{1}{\rho}=\frac{2\mathrm{TQ}}{\mathrm{PQ}^2}$$

But if PN is the projection of PQ parallel to the axis in (1), $PQ = PN$ approximately, and TQ in (2) = TQ in (1).

$$\therefore \frac{1}{\rho} = \frac{2TQ}{PN^2} \text{ in (1),}$$

$$\text{and acceleration} = \frac{2TQ}{PN^2} \cdot \frac{T}{m}.$$

$$= \frac{2}{PN}\left(\frac{TN}{PN} - \frac{QN}{PN}\right)\frac{T}{m}. \quad . \quad . \quad . \quad (1)$$

Here $\frac{TN}{PN}$ is the tangent to the angle of slope at P, and is the value to which $\frac{QN}{PN}$ approaches as Q is brought up to P.

From the equation to the curve we have—

$$QN = a \sin \frac{2\pi}{\lambda}(x + PN) - a \sin \frac{2\pi x}{\lambda},$$

$$= a \sin \frac{2\pi x}{\lambda}\left(\cos \frac{2\pi PN}{\lambda} - 1\right) + a \cos \frac{2\pi x}{\lambda} \sin \frac{2\pi PN}{\lambda},$$

$$= -2a \sin \frac{2\pi x}{\lambda} \sin^2 \frac{\pi PN}{\lambda} + a \cos \frac{2\pi x}{\lambda} \sin \frac{2\pi PN}{\lambda},$$

or substituting small angles for sines—

$$= -\frac{2\pi^2 PN^2}{\lambda^2} a \sin \frac{2\pi x}{\lambda} + \frac{2\pi PN}{\lambda} \frac{a \cos 2\pi x}{\lambda},$$

$$\text{whence } \frac{QN}{PN} = -\frac{2\pi^2 PN}{\lambda^2} a \sin \frac{2\pi x}{\lambda} + \frac{2\pi}{\lambda} a \cos \frac{2\pi x}{\lambda} \quad . \quad (2)$$

As Q moves up to P, the first term in the value of $\frac{QN}{PN}$ vanishes, and

$$\frac{TN}{PN} = \frac{2\pi}{\lambda} a \cos \frac{2\pi x}{\lambda} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

subtracting (2) from (3) and substituting in (1)

$$\text{Acceleration} = \frac{4\pi^2}{\lambda^2} \frac{T}{m} a \sin \frac{2\pi x}{\lambda}$$

$$= \frac{4\pi^2}{\lambda^2} \cdot \frac{T}{m} \cdot y \quad . \quad . \quad . \quad . \quad . \quad (4)$$

If the string is now released, this gives the acceleration with which each particle moves, and since it is proportional to the displacement y of each, in the first moment of time each moves a distance proportional to y, and each ordinate is reduced in the same ratio, say $n : 1$.

But
$$y = na \sin \frac{2\pi x}{\lambda}$$
is still a curve of sines, and since the amplitude only enters through y, the new acceleration is still expressed by (4). The velocity acquired in the moment of time is also proportional to y, so that during the next moment all the ordinates are again reduced in one proportion, and the velocities will be increased in one proportion, and so on during all successive moments.

Hence the curve always remains a curve of sines of the same wave-length, the amplitude only changing; and, since the acceleration at any point is always expressed by
$$\frac{4\pi^2}{\lambda^2} \frac{T}{m} . y = \mu y, \text{ say,}$$
then if every point of the string executes a simple harmonic motion, the time of vibration will be—
$$P = 2\pi / \sqrt{\mu} = \sqrt{\frac{\lambda^2 m}{T}} = 2l \sqrt{\frac{m}{T}} \quad . \quad . \quad (5)$$
and the frequency will be
$$n = \frac{1}{2l} \sqrt{\frac{T}{m}} \quad . \quad . \quad . \quad . \quad (6)$$
If the time in a simple harmonic motion is reckoned from the instant at which the displacement has its maximum value a, the displacement at time t is
$$a \cos \frac{2\pi t}{P},$$
where P is the period, while the velocity is
$$-\frac{2\pi a}{P} \sin \frac{2\pi t}{P}.$$

Applying these formulæ to the string, which has amplitude $a \sin 2\pi\, x/\lambda$ when released at $t = 0$, then at t the form and the velocity of any point are given by—
$$y = a \cos \frac{2\pi t}{P} \sin \frac{2\pi x}{\lambda} \quad . \quad . \quad . \quad . \quad (7)$$
$$v = -\frac{2\pi a}{P} \sin \frac{2\pi t}{P} \sin \frac{2\pi x}{\lambda} \quad . \quad . \quad . \quad (8)$$
where
$$P = 2l \sqrt{\frac{m}{T}}.$$

We draw from (8) the important conclusion that the velocity at any time can also be represented by a curve of sines, with the same nodes but a different amplitude.

Let us now suppose that the string is initially, not merely released, but also projected, each point with a velocity represented by a curve of sines, so that—

$$y = b \sin \frac{2\pi x}{\lambda}$$

$$v = -c \sin \frac{2\pi x}{\lambda}$$

represent the displacement and the velocity. Equating these to the values in (7) and (8), we find at once to what amplitude of vibration they belong, for—

$$b = a \cos \frac{2\pi t}{\mathrm{P}},$$

$$c = \frac{2\pi a}{\mathrm{P}} \sin \frac{2\pi t}{\mathrm{P}}$$

giving

$$a = \sqrt{b^2 + \frac{c^2\mathrm{P}^2}{4\pi^2}}.$$

To this point we have supposed the string to form a single loop; but all the preceding investigation will apply to the motion of any one of a number of loops, so long as they have nodes at the ends, and we get the general result, that if a string be initially displaced into a curve of sines having p loops, and be projected with velocity represented by another sine curve with p loops, each having nodes at the ends, it will continue to move in a sine curve and with frequency—

$$\frac{p}{2l}\sqrt{\frac{\mathrm{T}}{m}}.$$

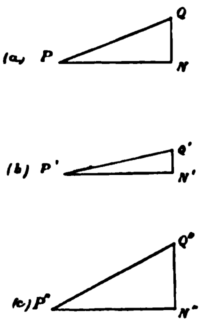


FIG. 44.—(*a*) An element of the string, with projection PN along the axis when displaced into a certain harmonic; (*b*) the same element with equal projection along the axis when displaced into another harmonic; (*c*) the same element with equal projection along the axis when having the sum of the displacements (*a*) and (*b*).

Superposition of Small Harmonic Vibrations.—Suppose, now, that two harmonic displacements are superposed. If the force due to the sum of the displacements is equal to the sum of the two forces due to the separate displacements, the acceleration is equal to the sum of the accelerations; and so the subsequent motion is the sum of the two motions, and the systems can be superposed.

Let PQ, Fig. 44, *a*, represent a displaced element in one harmonic, with projection PN parallel to the axis. Since the displacement and slope are both exceedingly small, P and Q remain sensibly in the same ordinate throughout the displacement, and PQ = PN. The force acting across the section at P, perpendicular to the axis, is

$$T \cos PQN = T \cdot \frac{QN}{QP}$$
$$= T\frac{QN}{PN} \quad \text{approximately.}$$

Let P′Q′, Fig. 44, *b*, represent the same element in another harmonic displacement. Then P′N′ = PN, and the component of the force at P′ perpendicular to the axis is

$$T \cdot \frac{Q'N'}{P'N'} = T\frac{Q'N'}{PN} \cdot$$

The sum of the force is, therefore,

$$T\frac{QN + Q'N'}{PN}.$$

But if P″N″, Fig. 44, *c*, represents the element when having the sum of the two previous displacements, the force across P″ perpendicular to the axis is

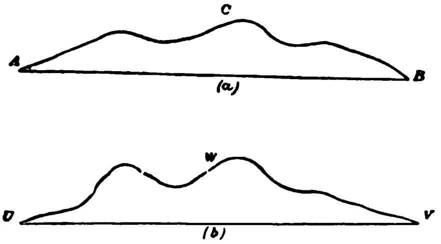


Fig. 45.—(*a*) Displacement and (*b*) Velocity of a String of length, AB = UV = l.

$$T \cdot \frac{Q''N''}{P''N''} = T\frac{Q''N''}{PN} \cdot$$

but Q″N″ = QN + Q′N′.

Hence, the total force at each end of an element, resolved in the direction of motion, is always the sum of the two separate forces, and therefore the motions may be superposed.

Clearly, also, any number of small motions may be superposed, so long as we may make the approximation PQ = PN.

We can now give an account of the most general transverse motion possible for a string subject to the conditions of fixed ends and small slope and displacement.

Let the curve ACB, Fig. 45, *a*, represent the curve of displacement initially given to a string of length l, and let UWV, Fig. 45, *b*, represent the curve of velocities of projection, both being really small. Then, applying Fourier's theorem to the two separately, each may be analysed into a series of harmonics of half-lengths, $l, l/2, l/3, l/4, \ldots$ and so on.

If any harmonic appears in both series, we must suppose that the string is not only displaced, but also projected with a velocity represented by the velocity harmonic. If a harmonic appears in the first series only, the string is only displaced; if in the second series only, the string is only projected for that harmonic. Since the motions are all superposable, we may consider each as going on separately, according to the laws we have investigated, in periods submultiples of $2l\sqrt{m/\mathrm{T}}$, and we may build up the motion at any future instant out of all the components. In the time $2l\sqrt{m/\mathrm{T}}$, all the harmonics will go through a whole number of vibrations, and the string will return to the same state. Further, since only one Fourier analysis is possible, and this solution always constitutes a Fourier series, no new harmonics can ever be introduced.

All this corresponds exactly with the results obtained by experiment, that a string may vibrate as a whole or in any whole number of loops, emitting notes of frequencies 1, 2, 3, . . . and that the compound note given out by the string in general consists of a definite mixture of harmonic tones.

The Overtones not always Harmonics.—The string considered in this investigation departs somewhat from the actual string, which necessarily possesses some amount of resistance to flexure, and which is more or less constrained in direction at the ends. When these conditions are taken into account, it can be shown that the overtones do not form exactly a harmonic series. It is perhaps worthy of note that the extra force due to rigidity, and the constraint at the ends increases the frequency of the string, for while the mass is the same as in an abstract string, the restoring force is greater.

Another example of a string with overtones not forming a harmonic series is given by a string loaded in the middle. If the mass of this load is great compared with that of the string, the fundamental mode of vibration is as represented in Fig. 46, *a*, each half of the string being always straight. If M is the load, the restoring force for displacement y, is $2\mathrm{T}\cos \mathrm{NCA} = 4\mathrm{T}y/l$ nearly. The acceleration is $4\mathrm{T}y/\mathrm{M}l$, and the frequency is

$$n_1 = \frac{1}{\pi}\sqrt{\frac{\mathrm{T}}{\mathrm{M}l}}. \qquad (1)$$

The second mode of vibration is shown in Fig. 46, *b*, the mass M remaining at rest, and the two halves of the string vibrating. The frequency is

$$n_2 = \frac{1}{l}\sqrt{\frac{T}{m}}. \quad . \quad . \quad . \quad . \quad (2)$$

This is very large compared with n_1, since M/ml is large.

The third mode is shown in Fig. 46, *c*, the string dividing up into segments, so that the time of AC or DB as a uniform string equals that of CD with the load in the middle; and since CD must be very small compared with AC, the frequency is only slightly more than n_2.

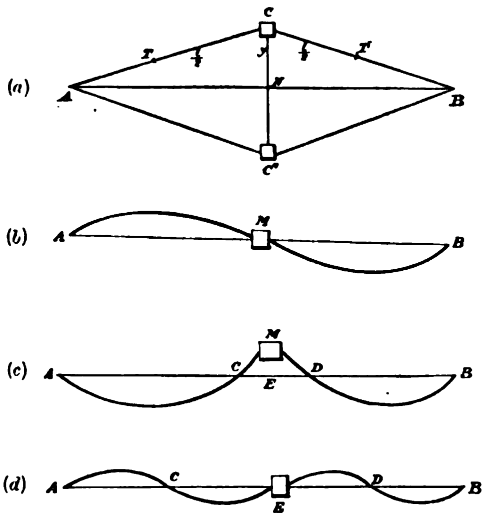


FIG. 46.—Modes of Vibration of a String loaded in the middle. The overtones occur in pairs at and near the even harmonics of the unloaded string.

The next mode is shown in Fig. 46, ***d***; and the next is when there are four loops, and a middle piece as in (*c*), the frequency being only slightly greater than in (*d*). Evidently the overtones occur in close pairs, the first members of the pairs forming a series, n_2, $2n_2$, $3n_2$, $4n_2$, &c. Taking the case of p loops, each of length l_1, and a middle piece of length l_2, we have

$$n_{p+1} = \frac{1}{2l_1}\sqrt{\frac{T}{m}}$$

also
$$n_{p+1}=\frac{1}{\pi}\sqrt{\frac{T}{Ml_2}}. \quad . \quad . \quad . \text{ from (1)}$$

and
$$pl_1+l_2=l.$$

Eliminating l_1 and l_2, we obtain a quadratic for n_{p+1}, of which the approximate solution is—

$$n_{p+1}=\frac{p}{2l}\sqrt{\frac{T}{m}}\left(1+4\frac{ml}{M}\cdot\frac{1}{\pi^2p^2}\right),$$

while
$$n_p=\frac{p}{2l}\sqrt{\frac{T}{m}}.$$

Second Method of Investigation—The Modes of Vibration Deduced from the Mode and Velocity of Propagation of Waves along a Stretched String.—The method is not so direct as the first, but as it is easy, and as it introduces us to some new and valuable ideas on the relation between waves and vibrations, it is worthy of attention.

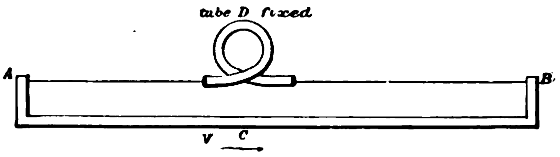


Fig. 47.—Framework moving String through fixed tube, D, with constant Velocity, V.

The Velocity of Propagation of a Disturbance along a Stretched String.—Let a string of mass *m* per unit length be drawn with uniform and constant velocity, V, through a smooth tube bent into a continuous curve of any form, and let the tension of the string be T. If this varied from point to point, we should have forces tending to accelerate the string in the direction of motion, and V could not be constant and uniform. Then T is also uniform throughout the length.

We may imagine as especially suitable for our subject an arrangement like that in Fig. 47, where a frame ACB moves in the direction AB with velocity V, drawing a string with the same velocity through a tube fixed at D. The ends are fastened to the frame at A and B, and the straight parts of the string are in one line. This limitation is not a necessity in the following investigation, but we may make it, since it is required in the application to musical strings.

The acceleration on any particle of string of small length, s, in the tube is altogether perpendicular to the curve, since the velocity along the curve is constant. Identifying the arc s with the same arc of the circle of curvature, its acceleration is evidently V^2/ρ, where ρ is the radius of curvature, and the force on it is $\dfrac{msV^2}{\rho}$.

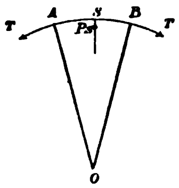


FIG. 48.—Element of string in tube. O centre of curvature.

But this force must be the resultant of the pressure of the tube against the string and the end tensions of the string itself. The former we may take as P per length 1 and + outwards. As to the latter, if AB, Fig. 48, represents the element s, and O is the centre of curvature, the tensions at the ends (which are equal to each other and perpendicular to AO, OB) may be represented by AO, OB. Their resultant is represented on the same scale by A B. Then this resultant is

$$\frac{T}{AO} \cdot AB = \frac{Ts}{\rho}.$$

We have, therefore, for the element s,

$$\frac{Ts}{\rho} - Ps = \frac{msV^2}{\rho},$$

whence

$$P = \frac{T - mV^2}{\rho}.$$

But if $V^2 = T/m$, $P = 0$, whatever ρ may be, and it vanishes throughout the tube at the same time. If we think of V as gradually increasing from zero, $V = \sqrt{T/m}$ is a critical value. Below this value the string presses inwards everywhere, and above it, outwards everywhere. But exactly at this value there is no pressure either way, and the tube ceases to be a constraint or guide. It may therefore be removed, and so long as the velocity has the critical value, the string will preserve the exact form of the tube in a fixed position, successive portions of the string moving into the curve as they come up to it.

But now let us change the standard of reference for the velocity of the string from the tube or whatever supported the tube to the frame itself. The ends and straight parts of the string are at rest, and the curve moves with velocity Vc, and if $V = \sqrt{T/m}$, it will move without constraint. This is, therefore, the velocity of propagation of a disturbance or a wave on a stretched string.*

---

* The reader may easily obtain this velocity by a method corresponding to that used for the velocity in air on p. 17.

An extension of this theorem to a case in which the tension and velocity vary from point to point may easily be roughly verified. Let one end of a uniform rope or cord—an india-rubber cord succeeds well—be fixed at some height, and let the other end hang down free. If x is the length below any point P, the tension at P is mgx, and the velocity of propagation of a disturbance through P is

$$V = \sqrt{T/m} = \sqrt{mgx/m} = \sqrt{gx}.$$

But comparing with the formula $v^2 = 2al$, we see at once that this is the velocity of a point starting from rest at the free end, and moving up the string with acceleration $g/2$. When the disturbance reaches the top of the string, experiment shows that it is reflected, and our formula tells us that, after reflection, it moves down the string with velocity $\sqrt{gx}$ and retardation $g/2$. At the lower end it is reflected again, and so on. If, for instance, the last few inches of the string or cord are bent up and allowed to fall, the disturbance can be seen travelling to and fro along the string, especially if the observer looks up along the string. The time of passage is $\sqrt{4L/g}$, where L is the length of the string; and if L is 8 feet, the time is approximately one second.

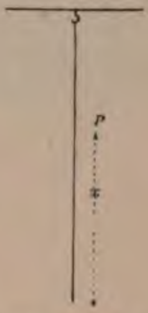


FIG. 49.—Uniform cord with lower end free transmitting waves with acceleration, $g/2$.

Superposition of Small Disturbances.—By the superposition of small disturbances, we mean that such disturbances may travel either together or in opposite directions, so that the disturbance at any point is always the sum of the separate disturbances which would exist at that point, supposing each to exist alone in an otherwise undisturbed system. The condition for this is that the acceleration due to the forces brought into play by the sum of the disturbances is equal to the sum of the accelerations due to the forces brought into play by each separately.

When this is true, evidently the motion throughout will be the sum of the separate motions, and they will be "superposed" without modifying each other in their subsequent history.

In the particular case of a stretched string, if the disturbance never moves a point out of its ordinate by a sensible amount, and if the stretching of any element is negligible, so that the slope is always infinitesimal, the investigation on page 89 shows that the condition is fulfilled.

Superposition of Two Equal Trains of Waves Moving with Equal Velocities in Opposite Directions.—Let two trains of equal waves, starting from coincidence, move in opposite directions with equal velocities. Let the waves be periodic curves with equidistant nodes, and let the ordinates at equal distances on the two sides of a node be equal and opposite, so that the second half of each wave is an inverted reflection of the first half. Since the two trains have always moved equal distances on opposite sides from the position of coincidence, the two ordinates arriving at the original position of a node A, as shown in Fig. 50, are always equal and opposite, so that the point remains at rest.

If the curves representing the waves are each originally given by

$$y = a \sin 2\pi x/\lambda \quad . \quad . \quad . \quad . \quad (1)$$

and if their velocity is V, their forms, t seconds later, when they have moved to right and left distances Vt, will be respectively

$$y = a \sin \frac{2\pi}{\lambda}(x - \mathrm{V}t) \quad . \quad . \quad . \quad . \quad (2)$$

and

$$y' = a \sin \frac{2\pi}{\lambda}(x + \mathrm{V}t) \quad . \quad . \quad . \quad . \quad (3)$$

Adding the two

$$y + y' = \mathrm{Y} = 2a \cos \frac{2\pi}{\lambda}\mathrm{V}t \sin \frac{2\pi x}{\lambda} \quad . \quad . \quad . \quad (4)$$

which is always a harmonic curve with the same nodes and amplitude varying from $+2a$ to $-2a$ and going through all its values twice in time, λ/V.

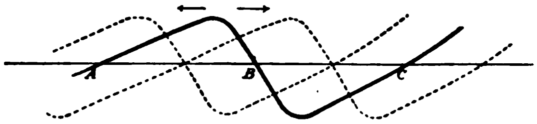


FIG. 50.—Two equal waves moving with equal velocities in opposite directions and always having equal and opposite ordinates at the nodes ABC.

The result of the superposition, therefore, is that the string is broken up into vibrating loops between nodes at rest, just as in Melde's experiments.

Considering any length $l = \frac{\lambda}{2}$, since the nodes are at rest we may fix them and cut off the outside parts of the string without affecting the included part, and we have a length l vibrating in time

$$t = \frac{\lambda}{\mathrm{V}} = \frac{2l}{\mathrm{V}} = 2l\sqrt{\frac{m}{\mathrm{T}}}.$$

But we may have waves of half the length with the same end nodes and one intermediate node. For these, $\lambda = l$ and

$$t = \frac{l}{\mathrm{V}} = \frac{2l}{2}\sqrt{\frac{m}{\mathrm{T}}}.$$

With waves of one-third the length and two intermediate nodes,

$$t=\frac{2l}{3}\sqrt{\frac{m}{T}},$$

and so on, forming a series of harmonic modes of vibration in times 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c., of the longest time.

All these modes may be superposed, and there is no necessary relation between their phases. We may, therefore, have any number of harmonics coexisting independently with times forming a series of submultiples of $2l\sqrt{\frac{m}{T}}$.

Given any initial displacement and distribution of velocities of a string fixed at both ends, Fourier's theorem enables us to express these as a mixture of harmonic displacements and velocities. Each of these harmonics may be considered independently, as we have just seen, and so we have a perfectly general solution for the vibration of a string.

Reflection of a Train of Waves at a Fixed Point—Stationary Waves.—We may regard a train of waves as containing a certain amount of energy which is transferred from point to point along the string with velocity V. When this begins to arrive at the end of the string it cannot pass out, for the tension cannot do any work on the fixed point, since this does not move. The energy, therefore, remains in the string, and a thorough mathematical theory shows that it is reflected, and that a disturbance travels back. Though such a theory is beyond the range of this work, we may take it as proved by experiment that such reflection does occur, and we may easily investigate its nature, assuming that it must satisfy the condition that the end remains at rest.

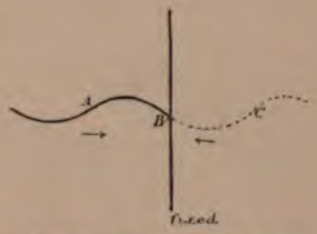


FIG. 51.—Reflection of a Wave, AB, at a fixed end. BC, the Reflection moving on to the String.

For let AB, Fig. 51, be an arch of a wave just beginning to arrive at B. If we imagine BC, an inverted optical reflection of AB, to be waiting, as it were, to come on to the string, and to move from right to left while AB moves from left to right, it will be seen that B always remains at rest. This is equivalent to starting a train of reflected waves moving in the direction BA, and coincident with the incident waves at the instant when a node of the latter reaches B. If the incident waves are continuous, the reflected waves will combine with them to form a series of vibrating segments with fixed nodes; and this is exactly what occurs in Melde's experiment (p. 83).

Now consider a string fixed at both ends and displaced into a sine curve as AGB, Fig. 52. We may think of this sine curve as consisting of two, AHB and AH'B, each of half the height of AGB and superposed. When the string is released AHB moves to the right and AH'B to the left. At once reflections occur against the fixed ends, that against B being represented by supposing BKC to move towards the left, and that against A by supposing AK'D to move towards the ~~left~~. Thus the reflections continue to supply the oppositely moving equal trains. This view of the matter shows us that, in regarding the vibrations of a string as made up of two opposite trains of waves, we are not merely adopting a mathematical device, but that these trains have a physical existence due to the end reflections. Such vibrations between fixed nodes, due to the superposition of direct and reflected waves, are frequently termed **stationary waves.**

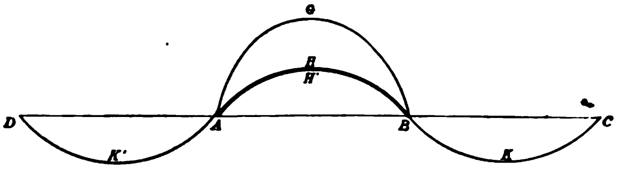


FIG. 52.—Resolution of a Displacement of a String, AGB, into two half as large, each moving in opposite directions and reflected at the ends.

The supposition underlying the foregoing investigation, that the reflecting end of a string is fixed, is only approximately satisfied. Some of the energy does leak out by the small movement really occurring, and the waves are communicated to the sound-board or framework of the instrument of which the string forms part. The energy of vibration, therefore, gradually dies away, and the reflected waves are not quite as large as the direct waves. Of course, this leakage through the ends is not the only one. Some leakage occurs through bending the string against the "viscous" part of the resistance to flexure, and some energy goes directly into the air. But inasmuch as many hundreds of vibrations may succeed each other without serious diminution of amplitude, the results obtained, when we entirely disregard the leakage of energy, are a very fair approximation to the truth.

CHAPTER VII.

PIPES AND OTHER AIR CAVITIES.

CONTENTS.—Sounds from Pipes closed at one End or open at both Ends—Modes of Vibration of Pipes—Pipes used as Musical Instruments—Reed Pipes—The Velocity of Sound in Pipes—Kundt's Dust-Tube—Pressure-change, and Motion in Organ Pipes—Sounds from other Air Cavities—Vibrations of Liquids in Pipes.

IN a large number of musical instruments the vibrating source of sound is the column of air in a pipe. The shape of pipe and the mode of excitation vary greatly, but the general laws of vibration are approximately the same in all cases. We shall, therefore. begin with an account of an experimental investigation, which may be easily made in the laboratory, of the mode of vibration of the air in circular tubes when excited by resonance. The abstract nature of the case, that is, its freedom from the complications necessary in practical instruments, will not be found to render the laws obtained inapplicable to such instruments.

Pipe Closed at One End.—A convenient form of pipe closed at one end consists in a glass tube of circular section, say 50 cm. long and 2 or 3 cm. in diameter, held vertically, with the open end upwards, and the lower end connected, as indicated in Fig. 53, with a reservoir of water. This reservoir can be moved up and down so as to vary the level of the water-surface in the pipe and at the same time to vary the length of the column of air. A vibrating tuning-fork is held with one prong over the open end to throw the air into vibration. The forced vibrations executed in unison with those of the fork are in general feeble, though quite audible; but if the water-level be varied by raising or lowering the reservoir, at one particular level the sound increases

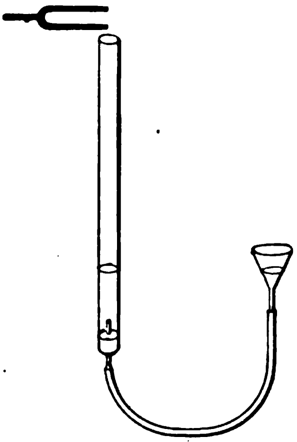


FIG. 53.—Pipe open at the upper end, closed at the lower end by a water-surface, of which the level may be varied by raising or lowering a water-cistern connected by flexible tube. The vibration excited by a fork. Resonance where the length of the air column is a quarter wave-length of the fork.

enormously. The pipe is then of such length that its own frequency is equal to that of the fork. In fact, the pipe resounds to the fork. The fork by itself in open air can only slightly affect the air, and so it only slowly loses its energy. But if it is held over a column of air in unison with it, it can give up its energy comparatively rapidly to the column; which, in turn, gives out its energy to the surrounding air in the form of large sound-waves.

When experiments are made with pipes of different diameters, it is found that the length of pipe resounding to a given fork is nearly, but not quite, constant, the length diminishing as the diameter increases. It is not very wide of the mark to take as the result of experiment that, for a given frequency, length + 0·6 radius is constant.[*] We may term this the corrected length.

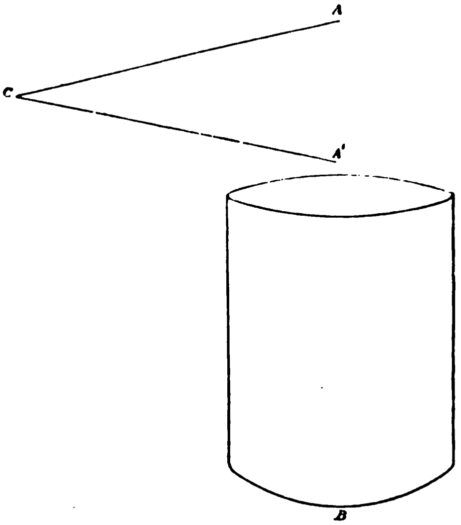


FIG. 54.—CA, CA', extreme positions of the prong of a fork exciting resonance in the pipe closed at B.

If experiments are made with different forks, it is found that the corrected length resounding to a fork is inversely as the frequency of that fork, and that it is one-fourth of the wave-length in air of the tone emitted by the fork.

A general explanation of this relation may be given as follows:—Let CA, CA′ (Fig. 54), be the extreme positions of the nearer prong of the fork. As it starts from CA towards CA′ it compresses the air in front of it, and the compression moves down the pipe. When it reaches the lower end, B, the air rushes down against the end and rebounds, so that the lowest layer rushes up and pushes against the next lowest layer; this pushes against the next, and so on. This means that the compression is reflected as a compression. If the length of the pipe is $\lambda/4$, the total distance traversed down and up again is $\lambda/2$. When, therefore, the front part of the compression gets back to the open end, the fork will just be at CA′ ready to return. The rush out of air which forms the compression will thus agree with the motion of the fork. Considering now the second half of the vibration, in the motion of the prong from CA′ to CA a rarefaction is sent down the tube. But the previous compression, which is meanwhile coming up to the open end, finds there a large space to expand into, viz., the open air, and so the air rushes out. But the outrush does not cease when the pressure falls to its normal value, for the momentum acquired continues the outrush. Consequently, there is a fall below the normal pressure, and this is propagated down the tube. In other words, the compression coming up is reflected back at the open end as a rarefaction, and the wave thus sent down agrees with that sent at the same time by the fork. It is easy to see that the rarefaction travelling downwards is reflected at the closed end as a rarefaction, and that it will come up to be reflected at the upper end as a compression agreeing with the compression which the fork is then sending down. The fork and the air in the pipe, therefore, agree in their motion, and the fork will continue to do work on the air and yield energy to it until all the air at the end of the pipe is moving through as great an amplitude as the prong of the fork.

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[*] Rayleigh, *Sound*, vol. ii. § 314.

We may here anticipate the result of a more thorough investigation in saying that an approximate representation of the motion of the air in the pipe is given by the up-and-down motion of a spiral spring hanging downwards from a fixed point, the point of support representing the closed end. In practice it is necessary to load the spring at the lower end, in order to reduce the frequency; but if we could make it vibrate slowly under its own weight alone, the analogy would be complete. As with the spring, so with the pipe, the whole of the air simultaneously closes in during one half of the vibration and opens out during the other half; and as with the spring there are the greatest variations of tension with the least motion at the fixed end of the spring, so there are the greatest variations of pressure with the least motion of the air at the closed end of the pipe. And as there is the greatest motion and least variation of tension at the lower end of the spring, so there is the greatest motion and least variation of pressure at the open end of the pipe. In a vibrating string the point of least motion is termed a **node**, and the point of maximum motion a **loop**. These names are usefully employed also to designate the points of minimum and maximum motion of a pipe. A closed end is therefore a node, and an open end a loop.

We may indicate the motion of the air as in Fig. 55 (1), where the arrow above the pipe shows the direction during the first half of the vibration, and that below, the direction during the second half.

The Overtones of a Pipe Closed at one End.—By holding over the open end forks of different frequencies or by varying the length for a given fork, it may be shown that the overtones are the odd harmonics having frequencies, 3, 5, 7, &c., times that of the fundamental tone. None of the even harmonics are present. These overtones are the fundamental tones of pipes

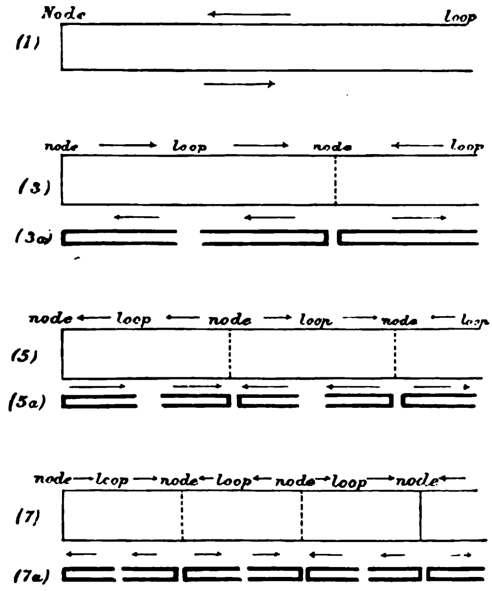

FIG. 55.—Modes of Vibration in the Fundamental Tone (1), and the harmonics (3), (5), (7), of a pipe closed at one end ; (3*a*), (5*a*), and (7*a*) show the smaller pipes into which we may suppose the full length to be broken up when the harmonics are sounding. The arrow-heads above show the direction of motion in one half, those below in the other half of a vibration.

having lengths $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$, &c., that of the pipe under consideration; and we may explain their existence by supposing the pipe to divide itself up into segments separated by nodes, each complete segment acting as two pipes with their open ends towards each other, as indicated in Fig. 55, (3*a*), (5*a*), &c. These nodes in the first harmonic are therefore $\frac{1}{3}$ and $\frac{3}{3}$ of the way down. In the next they are $\frac{1}{5}$, $\frac{3}{5}$, and $\frac{5}{5}$, and so on. It is easy to see that we can only break up the whole length into an odd number of equal pipes with one end closed, and so only the odd harmonics exist.

Pipes Open at Both Ends.—The fundamental tone of a pipe open at both ends has the same frequency as that of a pipe closed at one end of half the length. This may be shown by taking an open tube nearly twice as long as a given pipe which is closed at one end, and which resounds to a given fork, and putting a small sliding tube at one end, so that the length may be adjusted. The open tube will resound to the given fork when of about twice the length of the closed tube, and if the correction 0·6 radius be applied to each end of the open tube and to the open end of the other pipe, the corrected lengths are, within the range of experimental error, as 2 : 1.

We may represent the vibration of the air in the open pipe as like that in two pipes closed at one end and set with their closed ends together, as in Fig. 56 (1*a*), the air in both closing in towards or opening out from the closed ends at the same time in each. The motion of the air in the two halves of a complete vibration is shown by the arrows in Fig. 56 (1), the upper arrows for the first half, the lower ones for the second half. The air in the middle of the pipe is therefore always at rest, but is subject to variations of pressure. There is, consequently, a node there. At the open ends there are small variations of pressure but maximum motion, and the ends are loops.

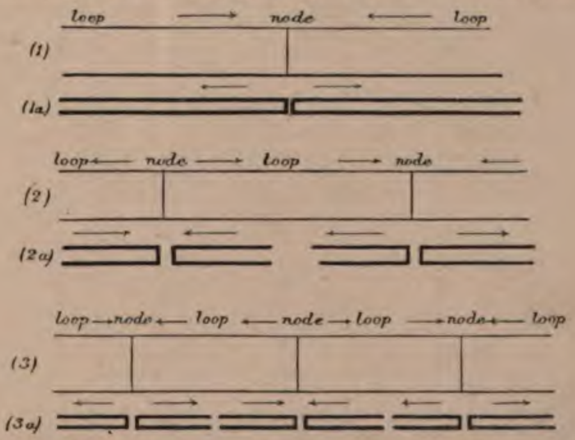

FIG. 56.—Modes of Vibration in the Fundamental Tone (1), and in the harmonics (2) and (3) of a pipe open at both ends; (1*a*), (2*a*), (3*a*) show the pipes closed at one end, into which we may suppose the full length to be broken up.

The Overtones of a Pipe Open at Both Ends.—Experiment shows that the overtones of such a pipe form a complete harmonic series, with frequencies 2, 3, 4 . . . times the fundamental frequency. Figs. 56 (2) and (3), with their decomposition into pipes closed at one end in (2*a*) and (3*a*), sufficiently show the nature of the vibration in the successive overtones, and evidently the process of decomposition can be carried on indefinitely.

Explanation of the Modes of Vibration in Pipes as Consisting of Stationary Waves Formed by the Superposition of

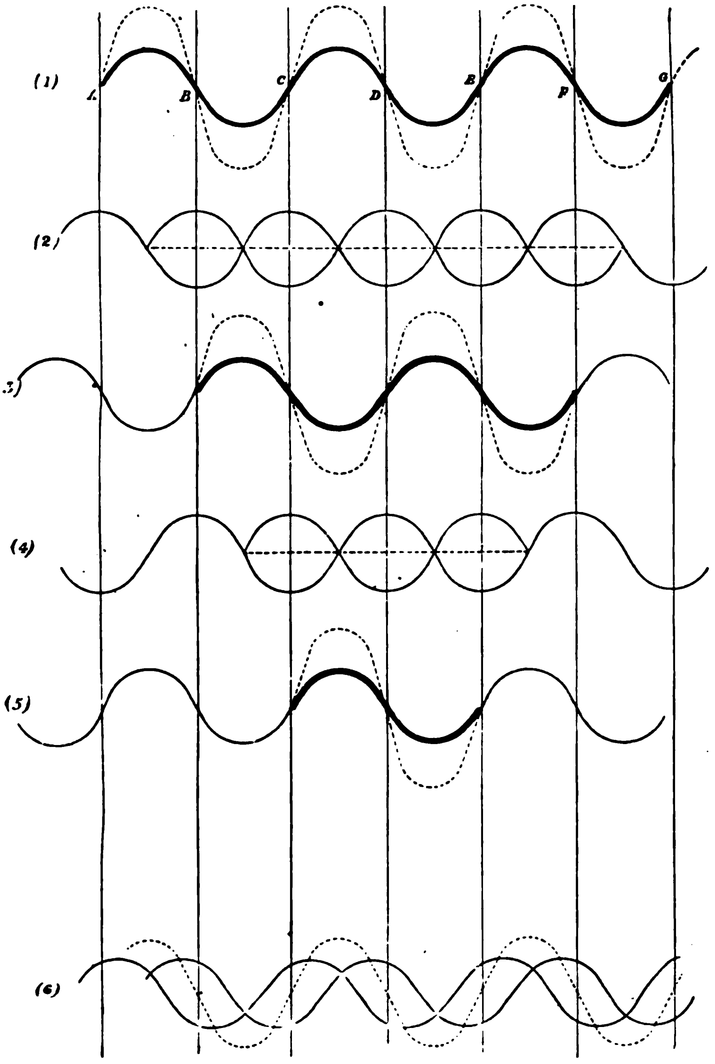

Fig. 57.—Formation of Stationary Waves by opposite motions of equal trains. (1) The two trains coincide, and each is represented by the black line; their resultant is represented by the dotted line. (2)–(5) Positions at successive intervals of $\lambda/4U$, the dotted line always showing the resultant. (6) A position intermediate between (1) and (2), showing that the nodes are always fixed.

Direct and Reflected Waves.—We may regard the various modes of vibration of the air in pipes, as consisting in the formation of stationary waves similar to those on strings, discussed in the last chapter, and due to the reflection of the disturbances at the open and closed ends.

Let us first consider the result of two trains of equal simple waves of wave-length λ, moving with equal velocities, each U, in opposite directions. Fig. 57 (1) represents three waves in each of the two trains at the instant of coincidence. The resultant displacement is represented by the dotted line; (2), (3), (4), and (5) represent the positions of the two trains at successive intervals of time $\frac{\lambda}{4U}$, and in each case the dotted line is the resultant displacement. The thick lines in (3) and (5) indicate the coincidence of the two trains. In (6) a position is taken intermediate between (1) and (2). An inspection of the figures is sufficient to show that at the points A, B, C, D, E, the air is always at rest, and that the displacement of the intervening air is always represented by a sine curve, which oscillates between the two extremes represented in (1) and (3). The time of a complete vibration is that occupied between (1) and (5) or $\frac{\lambda}{U}$. The two trains therefore form a series of stationary waves.

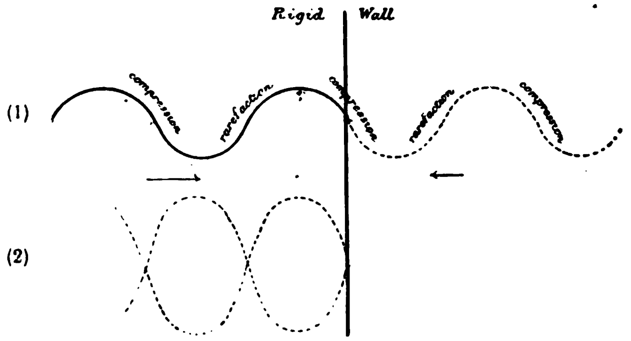

FIG. 58.—(1) Reflection at a Rigid Wall forming (2) a set of Stationary Waves with a Node at the Wall. The reflection may be supposed to occur by moving on the waves on the right in (1).

Such stationary waves may be formed in practice, in a manner similar to those on strings described on p. 96, by the superposition of a series of waves incident normally on a surface, and the waves reflected by the surface, when there is no loss of energy on reflection.

Thus when reflection occurs at a rigid wall, none of the energy can be transmitted onwards, and the reflection must be of such a kind that the layer of air immediately in contact with the wall remains at rest throughout. For if it moved from the wall, it would have a vacuum on one side and nearly atmospheric pressure on the other. The layer being thin and of exceedingly small mass, would thus be subject to a finite force and an exceedingly great acceleration. It would therefore at once be driven back on the wall. Now such reflection is obtained if we suppose that at the instant when a node of the incident waves reaches the wall, as in Fig. 58 (1), a series of waves, the continuation of the incident waves, is ready to move on in the opposite direction. The two trains will be just like those in Fig. 57, and will form a stationary system represented in Fig. 58 (2); the wall will always be a node. It is important to observe that compressions are reflected as compressions and rarefactions as rarefactions, as will be seen from Fig. 58, where the various parts of the waves are marked as compressions and rarefactions, and similar states are seen to be equidistant on the two sides of the wall. In other words, **there is no change of phase in reflection against a perfectly hard wall.**

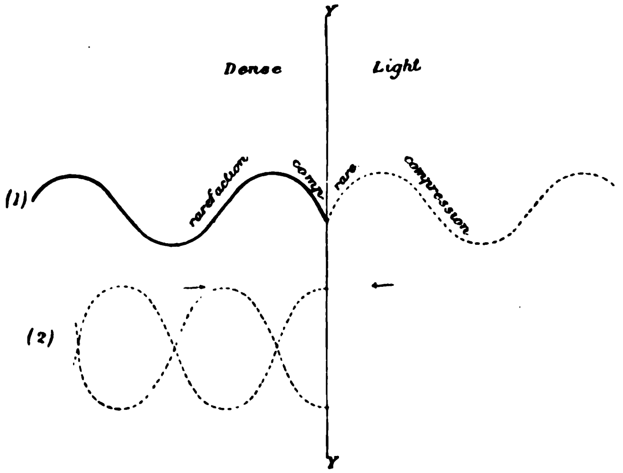

FIG. 59.—(1) Reflection at a Yielding Wall forming (2) a set of Stationary Waves with a Loop at the Wall. The reflection may be supposed to occur by moving on the waves on the right in (1), and it will be seen that this implies a change in phase at reflection.

But reflection may occur against a very yielding wall. For imagine that there is a surface separating a denser from a very much lighter medium, as in Fig. 59. When the variations of pressure come up from the left to the surface YY and begin to move into the new medium, they will almost disappear, for this medium being very much lighter, a difference of pressure on two sides of a layer of it will produce much greater motion. It moves, therefore, much more readily, and either makes room for the compression to expand, or itself expands and fills up the rarefactions, in each case bringing back the pressure towards the normal. But this reduction of pressure towards the normal implies reflection. This may be seen most easily if we take the extreme case of complete reduction to the normal, so that there is no pressure variation. For then the work done on the light medium during the outward motion is equal to that done on the heavy medium during the return motion. On the whole, then, no energy is sent across the separating surface, and the quantity returned must equal that arriving. Now, taking another point of view, consider a compression arriving at Y. The particles in a state of compression would, if the denser medium were continued, move a certain distance out and then return. But, as we have just seen, the first layer of the rare medium allows a much greater motion and goes much farther forward than the denser layers would do in the same place. It therefore draws away from the layers on the left, and forms a rarefaction which is propagated in the opposite direction, that is, backwards from Y. Similarly, when a rarefaction comes to the separating surface, the layer in front moves back much more readily than it would have done if it had been of the denser medium. The backward motion is, therefore, much greater, and so a backward motion of the particles is propagated back, *i.e.* a compression is reflected. **The reflection therefore occurs with a change of phase**, and the waves go back as if they had lost or gained half a wave-length by the reflection.

Taking the extreme case, in which the medium to the right is so light that it destroys all variations of pressure, and therefore reflects all the incident energy, it is easy to see that we obtain the condition of no variation of pressure by supposing that the series of waves on the right in Fig. 59 (1) are ready to move to the left when the waves on the left reach the separating surface. In Fig. 59 (2) the stationary waves so formed are represented. There is evidently a loop at the surface, and a node a quarter of a wave-length away.

This mode of reflection practically occurs at the end of an open pipe. For though the air outside is of the same density as that within, its freedom to move on all sides has the effect of reducing the pressure towards the normal value, the essential feature in the explanation just given. But the reduction is not complete, as it would be were the outside medium of less density. Hence the end of the pipe is not exactly a loop, but only near a loop. This necessitates the application of an "end correction," which may be calculated on certain suppositions.[1] The calculations are not, however, exact. Nor are the experimental determinations as yet satisfactory; but meanwhile we are not likely to be far wrong in taking the value for a circular pipe as a virtual addition to the length of 0·6 × radius.

We may now apply the foregoing investigation to explain the existence of stationary waves in pipes. Imagine a series of waves travelling from left to right in a pipe, Fig. 60 representing the displacement curve. Then, placing a rigid wall at any of the nodes, such as G, we shall obtain a reflected train moving from right to left, and forming with the incident train a system of stationary waves. G may represent the end of the pipe if closed. Or, if the end be open, we may place a yielding wall at any of the loops, such as P, and we again obtain a series of stationary waves. In order to continue to supply the series of waves from left to right, we may imagine another wall or closed end placed at any node to the left of G, such as F or E, or another yielding surface or open end placed at a loop, such as P, N, or M. The reflected train sent back from right to left will be again reflected from left to right, and so continue the incident train. Thus a pipe closed at one end and open at the other may have length from G to P, or from G to N, and so on, or $\frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots$; while a pipe open at both ends may have length from P to N, or from P to M, and so on, or $\frac{\lambda}{2}, \frac{2\lambda}{2}, \frac{3\lambda}{2}, \ldots$

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[1] Rayleigh, *Sound*, vol. ii. § 314.

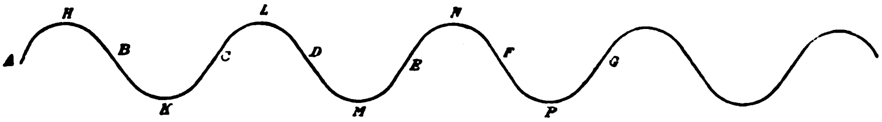

FIG. 60.

The time for a stationary wave to complete its vibration is equal to that taken by either of its constituents to move through λ, viz., λ/U.

If we keep the length of pipe l fixed, we see that the lengths of stationary waves which can be formed in it are—

For closed pipes	$\lambda = 4l,$	$\frac{4l}{3},$	$\frac{4l}{5},$ &c.,
with ~~frequencies~~ . . .	$\frac{4l}{U},$	$\frac{4l}{3U},$	$\frac{4l}{5U},$ &c.,
and for open pipes . .	$\lambda = 2l,$	$\frac{2l}{2}$	$\frac{2l}{3},$ &c.,
with ~~frequencies~~ . . .	$\frac{2l}{U}$	$\frac{2l}{2U}$	$\frac{2l}{3U},$ &c.

PIPES USED AS MUSICAL INSTRUMENTS.

The simple form of pipe which we have been discussing is most nearly approached in the instrument known as the Pandean Pipes, now merely a toy. It consists of a row of cylindrical pipes closed at one end, and with the open ends in a line. The player throws the air into vibration in any pipe by blowing across the open end. The vibration is probably maintained in the same way as in flue organ-pipes, which we shall now describe.

Organ-Pipes.—Figs. 61 and 62 represent respectively wood and metal pipes, the former material being used for square, and the latter for round ones. In each the base of the pipe is closed, but there is a slit through which the air is forced upwards from the wind-chest. In Fig. 61 the section of the pipe is exhibited exposing the various parts; *cd* is the slit, and just above it is an opening, the embouchure, virtually making the lower end an open end. The upper edge of this opening, *ab*, is sharp, and just over the slit. The upper end of the pipe may be open, an "open pipe," or it may be closed by an adjustable piston, a "stopped pipe." The "open" pipe corresponds to our abstract "pipe open at both ends;" the "stopped" pipe to the one closed at one end.

The mode in which the air in the pipe is set vibrating by the stream issuing from the slit is

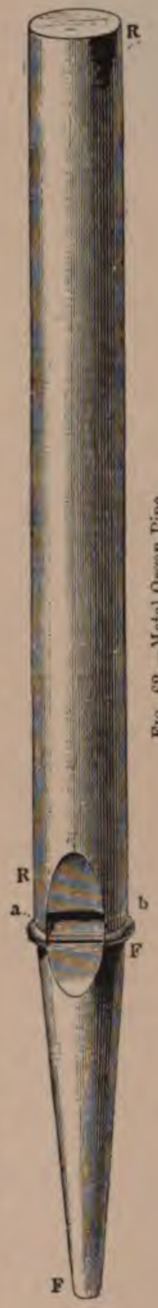

FIG. 62.—Metal Organ-Pipe.

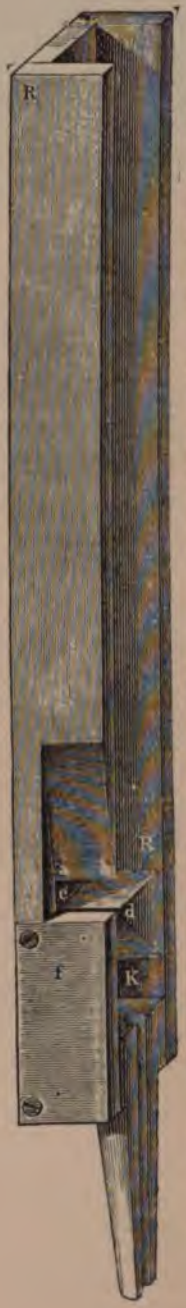

FIG. 61.—Wood Organ-Pipe in section. *K*, air cavity from which the air passes through the slit *c d* to impinge on the feather-edge *a b*.

probably somewhat as follows, though the explanation is by no means verified as yet. For simplicity we shall suppose the pipe "stopped" at the upper end. If the stream of air is directed to the inner side of the sharp edge, it will add to the quantity of air in the pipe, increase the pressure and so send a compression up towards the other end. If, on the other hand, the stream is directed to the outer side of the edge, it will rarefy the air in the pipe, just as a stream of air blown straight across the mouth of a tube tends to draw the air out of it. The draught up a chimney without a fire in a steady breeze is a case of such suction. The stream is probably so directed that a very slight disturbance will send it to either side of the edge.

Now let us suppose that during the first moment of blowing the air is accidentally directed within the pipe. A compression is formed and this travels to the stopped end, where it is reflected as a compression. It then travels back to the open end, where there is an outrush. Now even had the stream through the slit ceased, the outrush would have sent a rarefaction up the tube. But the stream is still flowing, and the outrush deflects it to the outside of the edge, where its suction action increases the rarefaction. When the rarefaction returns after its journey up the tube, there is an inrush of air from without, starting a compression. But with this inrush the stream is again brought inside the edge and the compression is increased. This process goes on, the successive compressions and rarefactions increasing until the work done by the stream of air on the pipe is equal to the energy radiated as sound. The time during which a complete wave, compression + rarefaction, issues from the pipe is evidently that taken by two journeys up and down the pipe, or the length of the pipe is $\lambda/4$.

The explanation for an open pipe is similar, but complicated by the fact that there is a change of phase in reflection at each end.

This explanation so far gives no account of the fact that the tone of a pipe rises slightly with the strength of the air stream. This is so well known that in some organs provision is made for blowing certain pipes with one definite pressure of air and no other. Nor does it show how it happens that when the velocity of the air stream is sufficiently increased the pipe ceases to sound its fundamental and the first overtone is the lowest tone given out. Perhaps when the stream of air across the open end of the organ pipe increases considerably in velocity, the suction action aids the conversion of a compression into a rarefaction, and the compressing action aids the opposite conversion so much that the open end approaches sensibly nearer the ideal of a perfectly yielding surface, and therefore, the "end correction" is diminished. As to the tendency to give the overtone with a high velocity, we may perhaps find an explanation in supposing a quasi-elasticity given to the stream as its speed rises, making its time of return after disturbance diminish. Certainly disturbances will be more quickly carried off, and the stream near its outlet will sooner resume its normal line of flow with a quick than with a slow velocity. It is possible then that, after a certain speed is reached, it is difficult for the fundamental tone to deflect the stream for the requisite time of a semi-vibration. The overtone is able to do it for the shorter time of its semi-vibration, and so continues to be reinforced while the fundamental tone dies away. This explanation must be taken merely as a speculation.

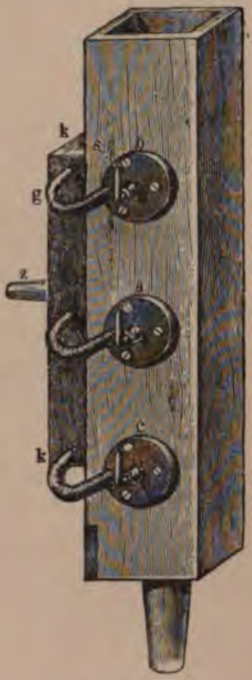

Fig. 63.—Kœnig's Organ-Pipe with Manometric Flames. *a*, node with fundamental tone; *b c*, nodes with first overtone.

It is necessary to have simple methods of tuning pipes when they are used in organs. With open pipes there is generally an opening in the side near the top with a tongue of thin metal which can be bent more or less over the opening, thus altering its size and the amount of communication with the outer air. With metal pipes it is sometimes sufficient to slightly alter the shape of the pipe, bending the top edge inwards to flatten, and outwards to sharpen the note. In many pipes there are side-plates, one on each side of the embouchure, and by bending these in or out the same effect is produced as by contracting or expanding the upper end.

Stopped-organ pipes are always closed by a piston fitting rather tightly in the upper end. They are tuned by moving the piston up or down.

The nature of the motion in an open pipe may be illustrated by lowering into the pipe a drumhead of paper stretched over a ring, with a little sand scattered over the paper. At a node, with no motion and only variation of pressure, the paper is still; but as the drum passes a node and moves towards a loop the motion increases, the paper is shaken up and down and the sand makes a rattling sound.

Kœnig has devised a very interesting mode of exhibiting the variations of pressure at the nodes by fitting three manometric flames (see p. 46) on to the sides of the pipe, as shown in Fig. 63, the membrane at one side of the gas chamber of each flame forming part of the wall of the pipe. When the pipe is sounding its fundamental tone, the middle point is a node with variations of pressure, and the flame at that point jumps up and down more than the other two. When the air blast is so far increased that the fundamental gives place to the first overtone, the middle point is a loop with no pressure variation, and the middle flame is still, while the other two, being at or near nodes, are greatly affected.

Reeds.—A reed consists of a tongue of metal fixed at one end and with the free end over an orifice, through which air is forced. The reed may overlap the orifice, when it is termed a **beating reed**, or it may be less than the orifice, when it is termed a **free reed** (Fig. 64). The wind blowing past the edges of the reed sets it in vibration, and so it alternately checks and allows the flow of air, and produces a regularly intermittent disturbance. In instruments of the harmonium and concertina class there is no accompanying pipe, the reed is alone concerned in the sound produced, the note depending chiefly upon its elasticity, material, and

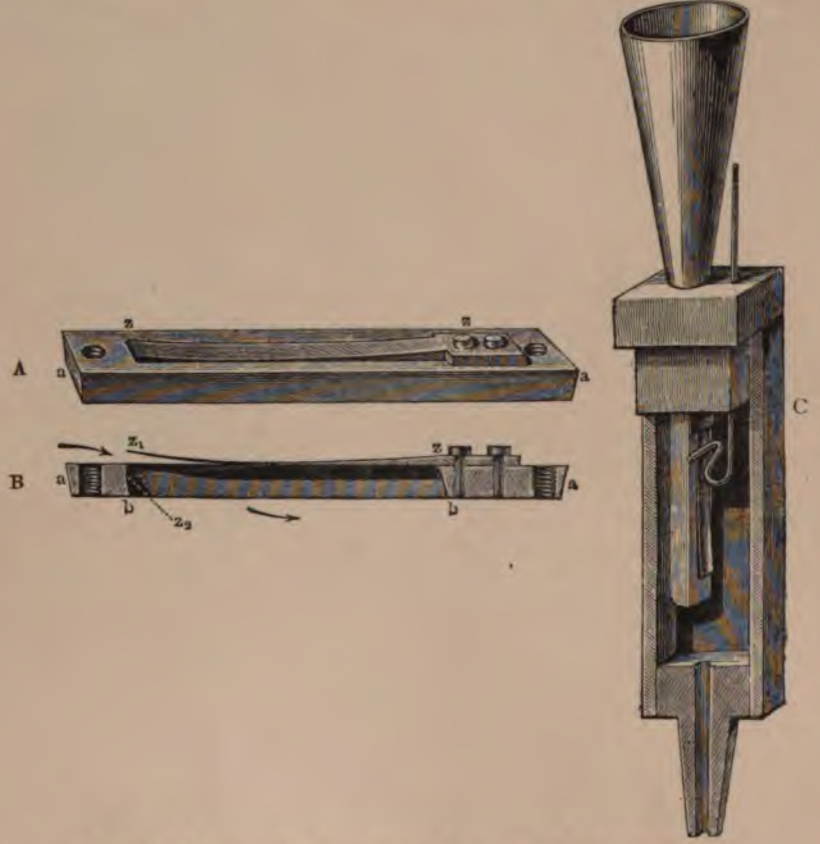

FIG. 64.—A. Free reed. B. Section of the reed. C. Reed as fixed in a pipe and provided with tuning wire.

form. But the orifice and stream of air are essential in the production of a loud sound, and doubtless modify the frequency of the reed.

In the organ, the reed is used in general in conjunction with a pipe to serve as a resonator reinforcing the tone given out by the reed. Tuning is effected by lengthening or shortening the vibrating tongue by moving a wire which presses hard against it. This is shown in Fig. 64, C.

The flute and flageolet are similar in their action to open organ-pipes. In the former, the player blows against the sharp edge of a round orifice in the side of the pipe, and in the latter the mode of excitation is exactly the same as in the organ-pipe. The length of effective pipe is varied by unstopping openings with the fingers or by opening valves.

There are several mouth instruments, as the clarionet and oboe, in which the reed is used; but in these it executes forced vibrations, being to a large extent, though not entirely, governed by the length of pipe. It may be regarded chiefly as emphasising the intermittent character of the stream of air which keeps up the vibration in the pipe, though not without some influence on the length producing a given note.

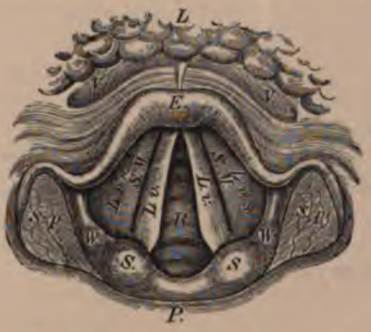

FIG. 65, A.—The larynx, as seen with the laryngoscope. *L*, the tongue; *E*, epiglottis; *V*, vallecula; *R*, glottis; *Lv*, true vocal chords; *SM*, sinus Morgagni; *Lvs*, false vocal chords; *P*, position of pharynx; *S*, cartilage of Santorini; *W*, of Wrisberg; *Sp*, sinus piriformes.

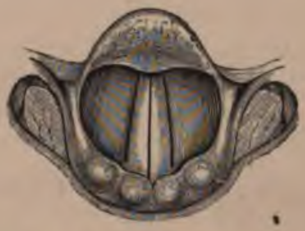

FIG. 65, B.—Position of the vocal chords on uttering a high note.

In mouth instruments, such as the trumpet and horn, the lips of the player serve as exciters of the vibration. They are stretched more or less tightly across the mouthpiece, and when the breath is forced through them, they each act as a kind of reed, forming together what we may term a **double reed**. The note they give varies with the tension to which they are subjected. The player can extract from the instrument either its fundamental or any one of a certain range of overtones by modifying the tension of the lips. In some instruments of this class the notes can be varied by valves in the side of the pipe. If one of these be opened, the effective length of the pipe is shortened and the fundamental tone and all the harmonics are raised in pitch.

The human voice-organ may be regarded as a double-reed instrument. In the larynx—the passage from the lungs—are two membranes, the vocal chords, just behind the projection in the throat known as "Adam's apple." These are attached one to each side of the larynx with a free straight edge across the tube (Fig. 65, A and B).

When we wish to sound the voice, we draw these membranes tight by a muscular effort, so that their edges are nearly parallel and nearly touching (Fig. 65, B). The larynx is thus nearly closed, and on forcing out the breath from the lungs, we make the vocal chords vibrate as a double reed and give out tones to which the cavities of the mouth and nose resound, and so the voice is produced.

THE VELOCITY OF SOUND IN PIPES

An approximate value of the velocity of sound in free air may be obtained by the experiment represented in Fig. 65, C, remembering that the length of the pipe is a quarter of a wave-length. For if λ is the wave-length, n the frequency of the note sounded,

$$n\lambda = U,$$

but $\lambda = 4L$, where L is the corrected length of the pipe. Then

$$U = 4nL.$$

But the velocity so obtained is not quite that of a wave in free air, but of a wave in the pipe used, and subject to the constraint and consequent friction of the sides. Both experiment and theory show that the velocity diminishes with the diameter of the pipe. This dependence came out very clearly in Regnault's experiments on the velocity in tubes (p. 28). Thus he found that the velocity in a tube 10·8 cm. in diameter was 324·25 m./sec., while in a tube 110 cm. wide it was 330·3 m./sec., tending towards an upper limit as the diameter increased. Several experimenters have sought to determine the relation between U and the diameter D. It will be sufficient

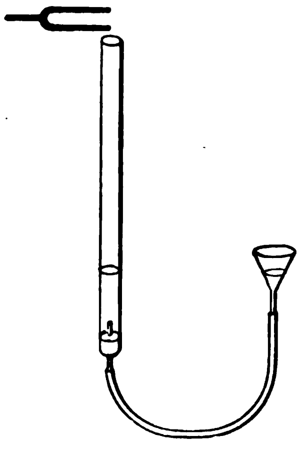
FIG. 65, C.—Pipe open at the upper end, closed at the lower end by a water-surface, of which the level may be varied by raising or lowering a water-cistern connected by flexible tube. The vibration excited by a fork. Resonance where the length of the air column is a quarter wave-length of the fork.

if we here refer to a research by Blaikley.[1] He used an organ-pipe with a pear-shaped cavity just above the speaking mouth, continued into a straight cylindrical pipe with a sliding plug in it. He first made the tube sound when the plug was in a given position, and then moved the plug outwards to such a position that the pipe could give the same note as before. The added length was the distance between two nodes or $\frac{1}{2}\lambda$.

Experimenting with several tubes, he found very satisfactory agreement with the formula already given on p. 28,[2]

$$U = U_0\left(1 - \frac{c}{D\sqrt{N}}\right),$$

where U_0 is the velocity in open air, D the diameter, and N the frequency. The most satisfactory value for U_0 he found to be 331·676 m./sec.

Kundt's Dust-Tube Method.—Kundt devised a very beautiful method of showing the mode of vibration on a tube closed at both ends, and of determining the velocity of air waves in such a tube. The apparatus is represented diagrammatically in Fig. 66. The "wave-tube," which must be very dry, has a little lycopodium dust scattered along it. It is closed at one end by a tight piston, *a*, and near the other end is a loose piston, *b*, at the end of a glass or metal rod—the sounding-rod. This is clamped firmly at its middle point, *d*, and on stroking it with a wet cloth if glass, with a resined leather if metal, it vibrates like the air in an open pipe, giving its fundamental tone, *d* being a node and *b* a loop. The piston at *b* communicates motion to the air in the wave-tube, and if the piston at *a* is carefully adjusted to some particular position, exact resonance occurs and a loud clear note rings out. The lycopodium dust is caught up by the air moving at the loops, and settles down where there is no motion, that is, at the nodes *nnn*. The process may be watched, for at each stroke of the exciting rod clouds rise from the loops, and the heaps collecting at the nodes increase. The velocity of sound in the material of the sounder being many times that in air, there may be many nodes in

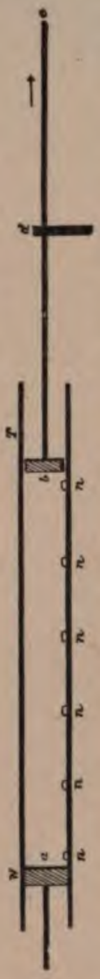

Fig. 66.—Kundt's Dust-Tube Method of determining Velocities in Tubes. WT, wave-tube closed by an adjustable tight piston at *a* and a loose piston at *b*. The loose piston is at the end of a glass or metal rod, *bc*, clamped at its centre, *d*, and excited by stroking in the direction of the arrow near *c*; *nn* are the heaps of lycopodium collecting at the nodes.

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[1] *Phys. Soc.*, vol. vi. 1884–5, p. 228.
[2] Rayleigh, *Sound*, vol. ii. §§ 347–350.

the wave-tube, the first being at *a* and the last near *b*. The piston *b* is the source of the energy, and communicates all the motion to the air. But the amplitude of vibration in the air soon far exceeds that of the rod, so that the motion of *b* corresponds to that of a point much nearer a node than a loop. By measuring the average distance between the nodal heaps the half wave-length in the tube is determined with some exactness, and thence $U = n\lambda$ may be found. Replacing the air by any other gas, the velocity in that gas may be determined.

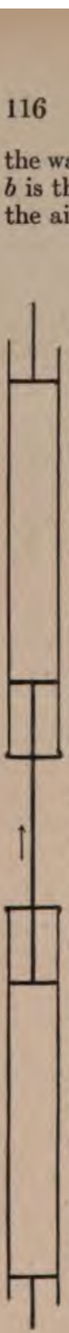

FIG. 67.—Kundt's Double Wave-Tube Apparatus, with one sounding-rod supported by india-rubber caps to the wave-tubes at the nodal points of its first overtone. The rod is excited by stroking it in the middle. The two tubes may be used for different gases.

In the first form of the apparatus the sounding-rod was clamped at its centre by the cork closing the end of the wave-tube. The vibrations were thus communicated to the solid walls of that tube, and not only were the appearances complicated, but the note was rendered slightly variable. Kundt finally adopted a form, represented in Fig. 67, with two wave-tubes containing different gases and a single sounding-rod. This was supported at $\frac{1}{4}$ and $\frac{3}{4}$ of its length, the nodal points for its first overtone, by passing it through india-rubber sheets covering the ends of the wave-tubes, and it was sounded by stroking it in the middle. The same note was of necessity sounded in the two tubes, and the india-rubber connection between sounder and wave-tube did not carry much vibration from one to the other.

Kundt found that for a given gas the velocity increased with the diameter of the tube and the wave-length, while it decreased with the roughness of the tube and an excess of lycopodium. He showed that the velocity was independent of the pressure and proportional to the square root of the absolute temperature. He also found relative values of the velocity in different gases, taking that in air as 1. Wüllner, using Kundt's method, and applying to his results Kirchoff's modification of Helmholtz's formula for the velocity in tubes, calculated the following velocities at 0° in free gas:—

	m./sec.
Air	331·90
Carbon monoxide	337·13
Carbon oxide	259·38
Nitrous oxide	259·64
Ammonia	415·99
Ethylene	315·90

Kundt and Warburg have used the double wave-tube to determine the ratio of the specific heats or γ in mercury vapour.[1]

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[1] *Pogg. Ann.*, clvii., 1876, p. 353.

One tube contained air, and the other, which was sealed, contained mercury vapour maintained at a high temperature in an oven. As dust to show the nodes quartz sand was used, and from the experiments the velocity U was determined. But (p. 20) $U^2 = \gamma P/\rho$. The temperature and the molecular weight of mercury gave P/ρ, and hence γ could be determined. It was found to be 1·66.

Experiments on Pressure-Change and Motion in Organ-Pipes.—Kundt[1] devised a water manometer, provided with a valve, which could be fixed at any point in the walls of a pipe. The valve opened outwards, and allowed a little air to escape into the manometer at each compression, until the pressure in the manometer was equal to the maximum pressure occurring within the tube at the point. The valve consisted of an india-rubber or gutta-percha membrane over a narrow slit in a metal plate. It was stretched so as not to be very different in tone, when blown as a reed, from the organ-pipe to which it was attached. This ensured its quick response to variations of pressure. By making the valve open inwards, the minimum pressure could be measured. Kundt found differences of pressure in a closed pipe 30 cm. long, amounting to 30 cm. of water on each side of the normal pressure, say 1/34th of an atmosphere. From this we may calculate the amplitude of the vibration, for the stationary wave in the pipe may be regarded as made up of two equal and opposite progressive waves, to which we may apply the investigation of Chapter II.

Let the amplitude in one of these waves be a, the maximum velocity of a particle in the wave u, the frequency n, and the variation of pressure p, while the velocity of sound-wave is U and the normal pressure is P. Then, if the particle moved round in a circle on its actual path as diameter with a uniform velocity equal to the maximum u, it would go round the circle in the time of vibration in the actual path. Then in one second it would travel

$$u = 2\pi n a$$

But (Chapter II. p. 18)

$$\frac{u}{U} = \frac{p}{E} = \frac{p}{\gamma P}.$$

Then eliminating u

$$a = \frac{U}{2\pi n} \cdot \frac{p}{\gamma P}$$

$$= \frac{4nl}{2\pi n} \cdot \frac{p}{\gamma P} = \frac{2lp}{\pi \gamma P}.$$

where l is the length of the pipe.

Using the values of the quantities given above, viz.—

$$l = 30,\ p/P = 1/68,$$

and putting $\gamma = 1{\cdot}4$, we find that at the open end of the tube

$$a = 0{\cdot}2 \text{ cm.}$$

---

[1] *Pogg. Ann.*, cxxxiv. 1868, p. 163.

We must double this for the stationary wave, and again for the whole swing. Thus the extent of vibration is nearly 1 cm.

Another method of investigating the range of pressure has been devised by Töpler and Boltzmann. This depends on the lengthening and shortening of the path of one of two interfering beams of light, and the consequent shifting of the interference fringes by the compression and rarefaction of the air in the pipe. In a particular case, they deduced an amplitude of 0·25 cm.[1]

Mach[2] investigated the excursion directly by the following method. An organ-pipe was fixed horizontally, and along the top wall on the inside ran a platinum wire previously wetted with sulphuric acid. When the wire was heated by an electric current, a fine line of vapour descended from each drop. The pipe was closed by a membrane at the centre to prevent any through draught of air, and when it was blown, the lines of vapour were carried to and fro; one of the side walls was of glass, so that the lines of vapour could be seen, and their extent of excursion measured by a stroboscopic method. The excursion at the end of an open pipe 125 cm. long was 0·4 cm.

Extent of Excursion in Waves Necessary for Audibility.—Lord Rayleigh[3] made an experiment on the amplitude of vibration in a sound just audible. The sound was produced by a whistle, of frequency 2730, blown by a measured current of air at observed pressure, the rate of working in producing the blast being

$$W = 1850000 \text{ ergs/sec. nearly,}$$

and it was certainly audible at a distance of 820 metres.

Now if v is the maximum velocity of a particle in a wave, it is easy to show that the average energy, kinetic and potential, per cubic centimetre, is $\frac{\rho v^2}{2}$.

If, then, it is assumed that the energy travels out in a hemisphere in the form of sound-waves, and that v is the maximum velocity of a particle 82,000 cm. from the source, the energy passing per second through the hemisphere of that radius is

$$W = \pi \times 82{,}000^2 \times 34{,}100 \times \cdot 0013 \times v^2,$$

where 34,100 is the velocity of sound at the observed density of the air, ·0013.

This gives $$v = \cdot 0014 \text{ cm./sec,}$$

But $$2\pi n a = v$$

where a is the amplitude and n is 2730.

---

[1] *Pogg. Ann.*, cxli.; Rayleigh, § 322*a*, 2nd ed.

[2] *Optisch-Akustischen Versuche.*

[3] *Proc. Roy. Soc.*, xxvi. 1877, p. 248; *Sound*, ii. § 384, 2nd ed.

Whence $$a = 8{\cdot}1 \times 10^{-8} \text{ cm.}$$

Another method of experiment carried out later gave

$$a = 12{\cdot}7 \times 10^{-8} \text{ cm.}$$

Other Air Cavities.—The term "pipe" is usually applied to a tube with a diameter small compared with its length. But musical notes may be obtained from cavities in which this condition is not fulfilled. Savart investigated the frequencies of cavities with rectangular section. He found, with cavities closed at the upper end, that if the embouchure runs along the whole of a lower edge, the length of that edge, or, in other words, the width of the side rising from it, has no effect on the frequency. This we might expect. He also found that the frequency was inversely as the square root of the area of the other vertical side, *i.e.* inversely as the geometric mean of the two edges other than the embouchure, so long as the height was less than six times the depth from front to back. When the height was more than twelve times the depth, the depth ceased to have any considerable effect, and the pipe might be regarded as an ordinary organ-pipe.

Cavaillé-Coll found that for open pipes of length l and depth p from front to back,

$$l = 2\lambda - 2p,$$

where λ is the wave-length.

For round pipes of diameter d he found that

$$l = 2\lambda - \frac{5}{3}d.$$

Savart found experimentally that for exactly similar cavities the periodic times are directly as the linear dimensions. This is a particular case of a general law applying to all vibrating systems in which stress is proportional to strain, and known as the **principle of dynamical similarity.** For consider two systems having linear dimensions in the proportion $l_1 : l_2$. Let them have similar displacements at corresponding points, *i.e.* displacements as $l_1 : l_2$. Taking elements of lengths as $l_1 : l_2$, these are altered in length in the ratio $l_1 : l_2$, and corresponding volumes therefore undergo equal strains. Hence, if the co-efficients of elasticity are as $E_1 : E_2$, the forces per sq. cm. at corresponding points are as $E_1 : E_2$. Now, thinking of similar parallelopipeds at corresponding points, the forces at the ends act over areas in the ratio $l_1^2 : l_2^2$, and therefore the total forces at corresponding ends are as $E_1 l_1^2 : E_2 l_2^2$. The forces at the opposite ends are in the same ratio, and therefore their differences producing acceleration are also in the same ratio. The masses are in the ratio $\rho_1 l_1^3 : \rho_2 l_2^3$; so that the accelerations are as

$$E_1 l_1^2/\rho_1 l_1^3 : E_2 l_2^2/\rho_2 l_2^3 = E_1/\rho_1 l_1 : E_2/\rho_2 l_2.$$

But these are for displacements as $l_1 : l_2$; for equal displacements they are as

$$E_1/\rho_1 l_1^2 : E_2/\rho_2 l_2^2.$$

The periodic times, being inversely as the square roots of the accelerations for equal displacements, are as

$$\frac{l_1}{\sqrt{\dfrac{E_1}{\rho_1}}} : \frac{l_2}{\sqrt{\dfrac{E_2}{\rho_2}}}.$$

In the case of air filling both systems, $E_1 = E_2$ and $\rho_1 = \rho_2$, and therefore

$$T_1 : T_2 = l_1 : l_2,$$

or the times of vibration are directly as the linear dimensions.

It may be noted that the times are proportional to the times taken by waves to traverse the two systems.

The modes of vibration of various air cavities have been investigated both theoretically and practically. The theoretical investigations in several cases will be found in Lord Rayleigh's *Sound*. The most important, practically, are those of resonators with and without necks, but the results are too complicated for us to consider here.

Another case of some interest is that of a rectangular closed cavity, say an ordinary room. Often there is a particular point in a room at which the sounding of a given note will lead to a long resonance. Among the various modes of vibration, each length has its own modes, as if it were a closed Kundt's tube, and possibly the point at which the resonance is most plainly heard may be one where the vibration can be excited in two directions at once.

The Vibrations of Liquids in Pipes complicated by the Yielding of the Walls.—In order to find the velocity of sound in a liquid, Wertheim immersed a pipe in a liquid and blew it with the liquid itself, obtaining an audible note. The formula applicable to air pipes gave for water a velocity U=1173 m./sec. at 11°, a value much below the true one. Kundt and Lehmann[1] applied the dust-tube method, using iron plugs, and they found that as the diameter decreased the velocity increased. This was doubtless due to diminished yielding of the walls in the narrower tube. The yielding quite vitiated the generality of Wertheim's result, and made the method valueless.

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[1] *Pogg. Ann.*, cliii. (1), 1874.

CHAPTER VIII.

RODS—PLATES—MEMBRANES.

CONTENTS.—Vibration of Rods—of Plates—of Bells—and of Membranes.

In treating of the vibrations of solids, it is necessary, except with strings, to take into account the elasticity of the vibrating material. In this chapter we shall consider some of the simpler cases of vibration of rods and plates, though, except in the single case first treated, we shall not enter into any mathematical investigation.

Longitudinal Vibration of Rods.—If a glass rod or tube, say 2 or 3 feet long, is clamped in a vice at one end, and stroked near the other end with a wet cloth away from the fixed end, it is easily made to vibrate longitudinally, giving out a loud and ringing note. If held or loosely clamped at a third of its length from the free end, it gives the twelfth of the fundamental note. In fact, by fixing the nodes properly, it may be shown that the modes of vibration are like those of an air-pipe closed at one end.

If a rod of double the length is clamped at its middle point, it gives the same fundamental note. If free in the middle, but clamped ¼ of the length from one end, it gives the octave, and, in general, its modes of vibration are those of an open air-pipe.

Similar vibrations may be obtained from metal or wooden rods by stroking them with leather powdered over with resin, or even with the fingers, if quite free from grease. A "woodharmonicon" has been constructed with rods projecting from a sounding-box, and of lengths giving the notes of the musical scale, but it is utterly without musical value.

We have the analogue of a closed air-pipe, such as the column in a Kundt's dust-tube, if we stretch a wire between two points and stroke it with a wet or resined cloth. It may be made to emit a fundamental note vibrating as one length, and, on damping, it gives harmonics. The notes are usually harsh in quality, but of easily recognised pitch, and follow the order of those of a closed pipe.

The frequency for a given material is, in all these cases, proportional to the length between contiguous node and loop, and is independent of the thickness, so long as it is small compared with the length. This suggests a speed of travel along the rod uniform and depending only on the material, and not on the cross section. We should expect such uniformity if the disturbance is longitudinal, and if the forces called into play are the ordinary forces investigated in the stretching or compression of a bar, those given by Young's modulus. For the force per unit area of the cross section depends on the stretching or compression, and if the cross section, and therefore the mass to be moved, be altered in any ratio, the force moving is altered in the same ratio.

It should be noted, however, that in stretching or compressing a rod there is a side contraction or expansion, which bears a ratio (known as Poisson's ratio) to the longitudinal expansion or contraction. This ratio cannot exceed $\frac{1}{2}$. For cork, it only slightly exceeds 0. Generally it is about $\frac{1}{4}$ or $\frac{1}{3}$, while in the case of jellies and india-rubber, in which the total volume change for moderate stretching forces is small, it is near the upper limit $\frac{1}{2}$. Hence, as a wave of longitudinal or axial disturbance travels along a rod it will be accompanied by radial disturbance. We can form an idea of the importance of this radial motion thus:—Let ADB,

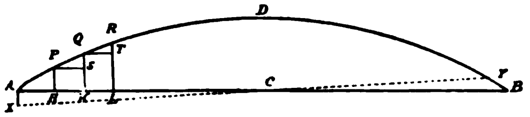

FIG. 68.—Displacement Diagrams for longitudinal and radial displacements in a rod. ADB longitudinal, XCY radial.

Fig. 68, be the longitudinal displacement diagram for a vibration of a bar, supposed round for simplicity. We may also suppose the curve to be a curve of sines. Take lengths AH, HK, KL, &c., each equal to the radius of the bar, and complete the diagram as shown; then PH, QS, RT, &c., represent the extension of lengths equal to the radius. If, then, σ denotes Poisson's ratio, the radial displacements of the outside of the rod will be σPH, σQS, σRT at the sections AH, HK, KL. These obviously decrease from A towards C, and drawing AX $= \sigma$PH, this is the amplitude of the lateral displacement at the surface. The displacement diagram can be shown to be a curve of sines of the same period as the longitudinal one. Clearly, then, if AH is a small fraction of AB, PH and *a fortiori* AX will be a small fraction of CD, and the kinetic energy of the radial displacement is negligible compared with that of the longitudinal displacement.

We shall now show that if we neglect the radial motion the velocity of propagation of a disturbance is

$$\sqrt{\text{Young's modulus} \div \text{density}}.$$

The mode of investigation is similar to that used in Chapter II., and we shall make similar assumptions, viz., (1) that the displacement is small compared with a wave-length, so that Young's modulus is constant (we shall see in the investigation how this is implied); and (2) that the displacement is the same for the whole of a cross section. We shall suppose a disturbance to be made and external forces applied in such a way that the disturbance is made to travel with uniform velocity, U, unchanged in form. Equating the force as given by the disturbance with the force as given by mass × acceleration, we shall find a value of U when the applied forces are all removed.

Let the curve HPQK (Fig. 69) represent the displacement. Let the external or applied force on the element MN of the rod per unit cross section be $X.MN$. The decrease in length per unit length at P is easily seen to be PS/ST. Then the force across the section at M per unit area is $M\cdot PS/ST$, where M is Young's modulus for adiabatic change. This is, for metals, nearly the same as for isothermal change. The force across the section at N per unit area is $M\cdot TV/VQ$ or $M\cdot RS/ST$. The resultant is $M\cdot PR/ST = 2M\cdot PR/MN$. The total force on MN per unit area is, therefore,

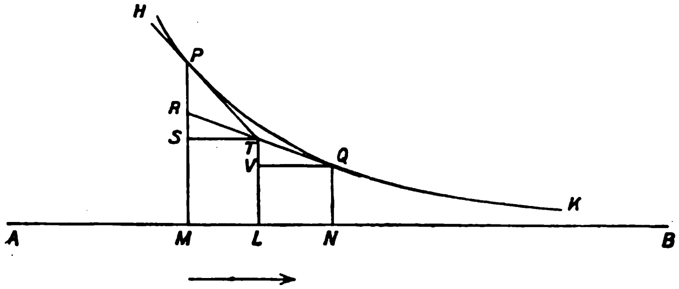

FIG. 69.—Displacement Curve for a longitudinal disturbance, all the ordinates being very small. TP, TQ, tangents at points so near each other that TP=TQ, and ML=LN.

$$X\cdot MN + 2M\cdot PR/MN \quad . \quad . \quad . \quad (1)$$

Now turning to the expression of force as rate of change of momentum. If the displacement changed uniformly in the time, MN/U, taken by the disturbance to travel from M to N, the ordinate QN would grow to MR. But it actually grows to MP, so that RP is the total displacement under the acceleration, which we denote by a.

Then
$$RP = \frac{1}{2}at^2 = \frac{1}{2}a \times MN^2/U^2$$

and
$$a = 2RP \times U^2/MN^2.$$

The mass-acceleration of length MN, cross section 1, and density ρ is

$$\rho a \mathrm{MN} = 2\rho \mathrm{RP} \times \mathrm{U}^2/\mathrm{MN} \quad . \quad . \quad . \quad (2)$$

Equating (1) and (2)

$$\mathrm{X} \cdot \mathrm{MN} + 2\mathrm{M} \cdot \mathrm{PR}/\mathrm{MN} = 2\rho \mathrm{PR} \times \mathrm{U}^2/\mathrm{MN},$$

$$\therefore \quad \mathrm{X} \cdot \mathrm{MN} = \frac{2\rho \mathrm{PR}}{\mathrm{MN}}\left(\frac{\mathrm{M}}{\rho} - \mathrm{U}^2\right),$$

and

$$\mathrm{X} = 0, \text{ if } \mathrm{U}^2 = \frac{\mathrm{M}}{\rho},$$

or

$$\mathrm{U} = \sqrt{\mathrm{M}/\rho}.^*$$

Since, then, a longitudinal wave travels along a rod or wire with constant velocity, we may apply the reasoning already used with regard to strings and pipes. Making two trains of equal waves move in opposite directions with equal velocities, we obtain a series of stationary waves. A fixed end of a rod or wire is a node, and a free end is a loop. We can then select the part of a system of stationary waves suitable to the system under consideration, the frequency of vibration being $\mathrm{U}/\lambda = \mathrm{U}/4l$, where l is the distance between a node and the next loop. The reader will easily see how to obtain the results if he has followed the reasoning in the case of pipes (Chap. VII. p. 108). Here there is no loop end-correction. For close to the free end the force must be indefinitely small, otherwise the acceleration of the last layer would be indefinitely great, since an entirely unbalanced finite force would be acting on an indefinitely small mass.

The observation of the frequency of the note emitted by a

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* It may be interesting to find the value of X when U has not this critical value, though the result is not applicable to Sound.

If r is the radius of curvature of the displacement curve at P, and if θ is the angle which it makes with RP,

$$\mathrm{RP} \cdot 2r \cos\theta = \mathrm{RQ}^2.$$

But according to our preliminary assumption, θ is nearly zero, and RQ is nearly equal to MN.

Then

$$\frac{2\mathrm{RP}}{\mathrm{MN}^2} = \frac{1}{r},$$

and

$$\mathrm{X} = \frac{1}{r}\left(\frac{\mathrm{M}}{\rho} - \mathrm{U}^2\right).$$

For instance, in a belt connecting an engine to a machine which it turns, we may regard the strained tight part and the unstrained loose part as the two portions of a wave going round the belt with velocity relative to the material equal to the circumferential velocity of either wheel. Then the frictional drag of the wheel against the belt must be such as to give a force per cubic centimetre equal to $\frac{1}{r}\left(\mathrm{U}_0^2 - \mathrm{U}^2\right)$, where U_0 is the unconstrained wave velocity and U is the actual velocity. From the analogy with forced oscillations we may term this a "forced wave."

given solid rod might be used to give us the velocity of sound in the rod: this is of little practical value. But since $M=\rho U^2$ and $U=n\lambda$, the same observation can be made to give a value of Young's modulus, which is of great practical importance. Wertheim in this way investigated the value of Young's modulus of many substances, finding it also directly for the same substances by loading wires and observing their extension. The two methods give nearly the same result, though the sound method gives always a slightly larger value, the excess being too great to be accounted for by the fact that in one experiment we are dealing with the adiabatic, and in the other with the isothermal modulus.

Kundt's dust-tube affords a pretty method of comparing the velocities in rods of different materials. The ends of the sounder being loops, its length is $\frac{1}{2}\lambda$ for the note emitted in the rod. The distance between consecutive dust-heaps in the tube is $\frac{1}{2}\lambda$ for the same note in air. Hence

$$\frac{\text{Velocity in rod}}{\text{Velocity in air}}=\frac{\text{length of rod}}{\text{length between dust-heaps}},$$

and comparing the lengths between the dust-heaps with different sounders, we obtain the ratio of the velocities.

Below are some values[1] for the velocity in different materials, obtained from Wertheim's determinations of Young's modulus, and the values for materials of the same name, obtained by Kundt's method.

Material.	Wertheim's Velocity in cm./sec.	Kundt's Velocity, air=1.
Brass . . .	355,000	10·87
Copper . .	375,000	11·96
Steel . . .	520,000	15·34
Glass . . .	520,000	15·25

It may be shown[2] that the velocity is diminished when the radial motion is too large to be neglected, and by $\frac{\sigma^2\pi d^2}{4\lambda^2}$ of the full value, where σ is Poisson's ratio and d and λ are the diameter and the wave-length. There is, therefore, a kind of dispersion of the constituents of a compound wave; so that in a "toy telephone," which consists of two drum-heads connected by a fine wire, one being spoken into by the sender, and the other being held to the ear by the receiver, the fundamental tone of each note will travel more rapidly than the harmonics. The slight practical importance of this will be realised by working out the case of a note of 1000 frequency in a brass wire 1 mm. in diameter (19 B.W.G.). It will be found that the slowing down is only one thirty-six millionth of the velocity for a very long wave.

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[1] Taken from Violle, tom. ii. pp. 75 and 193.

[2] Rayleigh's *Sound*, vol. i. § 157.

Torsional Vibrations.—If a rod or string be bowed perpendicularly to the axis with considerable strength, it may be made to vibrate torsionally, twisting to and fro round the axis. The torsional vibration of a monochord string may be rendered evident by putting a circular ring of paper on the wire, completed, after it includes the wire, by a piece of gummed paper. It turns rapidly round and round, first in one direction then in the other.

Torsional vibrations are usually of no practical importance. We shall not, therefore, investigate their frequency. The reader who has studied the torsional vibrations of wires in Elasticity will be able to adapt the general mode of determining wave velocity to this case. He must now take moments of the elastic forces and moments of the acceleration of momentum round the axis, and find their values in terms of the lengths on a displacement diagram with ordinates equal to the actual displacements of the outside of the wire. The velocity of propagation will be found to be $\sqrt{n/\rho}$, independent of the radius, where n is the co-efficient of rigidity. Hence the fundamental frequency of a wire of length l fixed at the ends is $\frac{1}{2l}\sqrt{n/\rho}$.

Transversal Vibrations of Rods.—The theory of the transversal vibrations of rods has been very fully worked out when the thickness in the direction of vibration is small compared with the length. The results of the theory have been very closely verified by experiment, which has in this case rather followed than preceded theory. The methods of investigation are far too advanced for this work, and we shall only quote the results. A full investigation will be found in Lord Rayleigh's *Sound*, vol. i. chap. 8. A simpler treatment is given in Violle's *Cours de Physique*, tom. ii. A. p. 195.

There are various cases, according to the mode of support or fixation, but we need here only take three of these.

Both Ends Free.—The harmonicon, which in civilised countries is merely a toy without musical value, affords an example of rods of this kind. It consists of a series of thin flat bars of iron, glass, or wood stretched across two more or less yielding supports, and the bars are struck with a yielding hammer. The frequencies of the tones emitted by such a bar are given by

$$n = \mathrm{UK}m^2/2\pi l^2,$$

where $\mathrm{U} = \sqrt{\mathrm{M}/\rho}$ is the velocity of longitudinal waves, K is the radius of gyration of the section, l is the length, and m has a succession of values, of which the first is $3{\cdot}011\frac{\pi}{2}$, and the rest are given to four figures by $5\frac{\pi}{2}$, $7\frac{\pi}{2}$, $9\frac{\pi}{2}$, &c. The nodes in the first three tones are in the positions given in the following table:—

Number of Nodes.	Positions in Fractions of Length from one End.	Frequencies.
2	·224, ·776	1
3	·132, ·5, ·868	2·76
4	·094, ·357, ·643, ·906	5·40

These tones can easily be obtained from a long flat bar, such as a very long bar magnet, and even from a wooden metre rule, by placing the supports at the nodes. In the latter case the tones have not much musical quality, but can easily be distinguished if the bar is tapped with the finger-nail.

Both Ends Fixed.—Apparently no musical instrument has been devised with such rods, and their only interest is theoretical. If the ends are fixed, so that their inclination cannot be changed (as when a rod is clamped at the ends in two vices), the frequencies of the tones emitted are exactly the same as for a rod of the length between the clamps and free at both ends—the case last considered. The first tone has no node, the second has one in the middle, and the third has two, respectively at distances ·359 and ·641 from one end.

One End Clamped, the Other Free.—Rods of this kind are used in musical boxes. The tuning-fork may also be considered as a pair of such rods clamped parallel. The free reeds used in some organ-pipes, and controlling their vibrations, are also clamped rods, though their period is doubtless affected to some extent by the air in the pipe.

The tones emitted are given by

$$n = \pi U K m^2/2l^2,$$

where m is $1{\cdot}194\frac{\pi}{2}$, $2{\cdot}989\frac{\pi}{2}$, or, to several figures, one of the series $\frac{5\pi}{2}$, $\frac{7\pi}{2}$, . . . The positions of the nodes are given in the table below.

Number of Nodes.	Positions in Fractions of Length from Free End.	Frequencies.
0		1
1	·226	6·25
2	·132, ·5	17·5
3	·094, ·356, ·644	34·3

These results may be roughly illustrated by clamping a knitting-needle in a vice and sounding it by drawing a fiddle-bow across a loop, a node being damped.

The very rapid rise in frequency of the overtones is of great practical importance in the case of tuning-forks. The first overtone of a fork with uniform prongs is more than $2\frac{1}{2}$ octaves above the fundamental tone, and the second more than 4 octaves above, and these die away more rapidly than the fundamental. The ear is, therefore, chiefly impressed by the fundamental tone. When the fork is held over a cavity resounding to this tone, the cavity is not likely to resound to the overtones as well, and so the predominance of the fundamental comes out still more strikingly. In practice there are greater or less departures from uniformity in the construction of the prongs of forks, and therefore the overtones are not quite those for uniform bars. Von Helmholtz found that in some forks which he examined the frequencies of the first overtones lay between 5·8 and 6·6 times the fundamental frequency.

Lord Rayleigh has pointed out that when a fork is mounted on a resonance-box the octave may generally be detected in the note given out by the box. This octave is due to the pull exerted by the prongs of the fork on the box, for they move in curved paths concave to the box, and therefore require a force towards the box to keep them in these paths. This force will be zero at each point of rest, and will go through the same series of values in each half vibration of the fork. In other words, its period is half that of the fork. The reaction is a pull on the box of the same period, and therefore producing the octave. In verification of this view, Lord Rayleigh mounted a 256 fork on a 512 box, and found that when the fork was powerfully bowed the octave was predominant.[1]

Temperature Correction for Forks.—A rise of temperature acts in two ways to alter the frequency of a fork, inasmuch as it increases the linear dimensions and decreases the elasticity. From the formula the frequency is proportional to

$$\frac{K}{l^2}\sqrt{\frac{M}{\rho}},$$

where M is Young's modulus, K is the radius of gyration, l is the length, and ρ the density. But with a given bar, ρ is inversely as the volume or l^3, while K varies as l. Hence

$$n \propto \sqrt{Ml}.$$

Now the co-efficient of expansion of steel is about $+ 1/10^5$, while the temperature co-efficient of Young's modulus is of the order of $- 2/10^4$. The latter, therefore, has by far the greater effect.

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[1] *Phil. Mag.*, (5) iii. p. 460.

M'Leod and Clarke, determining the frequency by the method of p. 43, found that the temperature co-efficient for a fork was given by

$$n_t = n_0(1 - \cdot 00011t).$$

The Vibrations of Plates.—From a musical point of view the vibrations of plates have interest, in that gongs and bells may be regarded as particular kinds of plates. The theoretical treatment is only possible as yet with flat or cylindrical sheets of metal, and is then sufficiently difficult. But since Chladni devised the simple mode of showing the nodal lines on round and square plates by means of sand, the great beauty of the figures formed by the vibration of plates have always had much interest for the experimenter.

Chladni's Figures.—To obtain these figures a uniform glass or metal plate, either round or square, is supported horizontally, usually by screwing it down to the top of an upright by a screw through its centre, but preferably by a clamp with cork lining to the jaws. This mode of suspension, while fixing the point clamped, does not fix the inclination of the plate, and so gives more freedom to the vibrations. To excite the vibrations, the edge of the plate is bowed by a violin-bow pressed hard against it and drawn vertically downwards. The point bowed is necessarily one of vibration. If a point on the edge or surface is touched by the finger-nail, the constraint is enough to fix it; and if a mode of vibration exists in which a nodal line, or line of rest, passes through the point touched while the point bowed is one of motion, that mode of vibration will perhaps be excited. If a little white sand

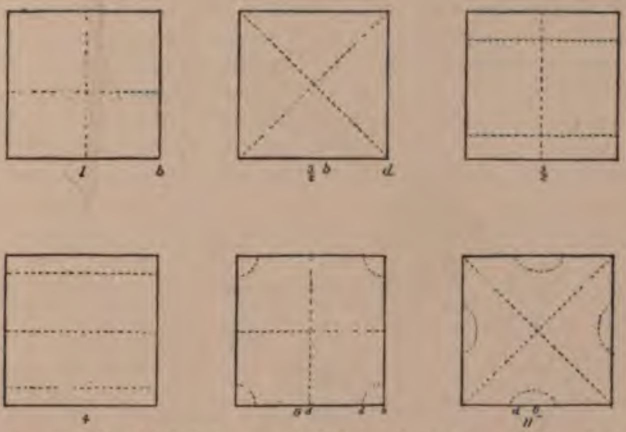

FIG. 70.—Simpler Chladni's Figures, with Approximate Frequencies. Points of bowing, *b*; and of damping, *d*.

has been previously scattered on the plate—the less the better—it is tossed from the areas of vibration on to the nodal lines, and a Chladni's figure is formed with surprising rapidity. A skilful bower who knows his plate may bring out a very great variety. Fig. 70 shows a few figures corresponding to the lower tones of a square plate. The relative frequencies are marked on each, and the points *b* and *d* are those of bowing and damping respectively.

With round plates there are two distinct types of vibration, respectively having nodal circles and nodal radii. To obtain the gravest mode of vibration with a nodal circle, it is sufficient to support the plate on the thumb and fingers at points about $\frac{2}{3}$ of the radius from the centre, and then to strike the centre of the plate with an india-rubber hammer. Sand scattered on the plate at once collects in a circle, which is the more sharply defined the more limited the supports and the more exact their position under the nodal line. Two nodal circles may be obtained in a similar manner, supporting the plate about $\frac{2}{5}$ of the radius from the centre and striking it with a harder hammer. But two and three circles may be more certainly obtained by supporting the plate on pointed corks under a nodal line and then rubbing a cord up and down against the side of a hole in the centre of the plate.

Experiment and theory agree approximately in assigning the positions of the nodal circles in the first three cases as in the following table:—

No. of Nodal Circles.	Distances from the Centre.	Frequencies.
1	·680	1
2	·392, ·842	4
3	·277, ·591, ·894	9

In the diametral mode of vibration the nodal lines are diameters, and successive sectors move in opposite ways. The lowest number of diameters is two, and there may be any greater number, the frequency of the tone emitted being proportional to the square of the number of diameters. With a plate clamped at the centre and sanded over, it is very easy to bring out the various figures by bowing and at the same time damping two consecutive nodal points on the edge.

There are also modes of vibration in which the nodal lines are both circular and diametral. The frequency in such a mode is proportional to $(d+2c)^2$, where d = number of nodal diameters and c = number of nodal circles.

In all these plate vibrations, adjacent segments separated by a nodal line move at a given instant in opposite directions. This may be verified by holding the hand or a specially shaped cover

over one segment of a pair. There is an immediate swelling out of the sound, due to the fact that the vibrations from the uncovered segment are no longer interfered with. When both segments are exposed to free air, one is sending forth a compression when its neighbour is sending forth a rarefaction, and the two practically neutralise each other at a little distance from the plate.

There is a curious fact with regard to Chladni's figures, first explained by Faraday. If lycopodium dust is mixed with the sand on a plate, the lycopodium collects in heaps at the middle of the vibrating segments, and not at the nodal lines. Faraday showed that this was due to eddies of air. They catch up the light dust and whirl it about where they are strongest, that is, over the points of greatest vibration. When the motion ceases, the eddies die away and drop the dust immediately below the region where they existed. In a vacuum the lycopodium gathers with the sand on the nodal lines.

Bells.—Bells are sometimes treated as if their modes of vibration might be deduced from those of flat circular plates vibrating with diametral nodes, the plates being supposed to be turned down at the edges till they are bell-shaped. But it is better to regard bells as modifications of cylinders, for this at once brings out the fact that the vibrations at the rim are both radial and transversal. For consider a thin, circular, cylindrical shell: the forces resisting extension are very large compared with those resisting bending, and the vibrations depending on them are very much higher, just as the longitudinal vibrations of a rod are much higher than the lateral ones. We may therefore regard the perimeter as bending only, and having a constant length.

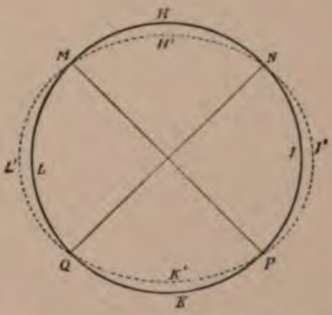

FIG. 71.—Section of Vibrating Cylinder, or Perimeter of a Bell. MNPQ, radial nodes and tangential loops; HIKL, radial loops and tangential nodes.

Taking the gravest mode of vibration with four nodes, represented in section in Fig. 71, the motion will be entirely radial at the middle points of the loops HIKL. But the nodes MNPQ are not points of rest, though points of no radial motion; for clearly MH′N is less than MHN, and ML′Q is greater than MLQ. There must therefore be tangential motion at the nodes to allow of these variations in length. The nodes, in fact, for the radial motion are loops for the tangential motion, and *vice versâ.*

A bell may be regarded as a vibrating cylinder of this kind pinched in at the top and bent out below, and it has just the same radial and tangential motions.

The mode of vibration of a bell-shaped glass jar may be illustrated very beautifully by filling it part way with methylated spirit and then by bowing the edge of the glass. The spirit which rises against the side of the jar is thrown into the centre, forming a cross of drops which do not at once coalesce with the body of the liquid. The appearance in two cases is indicated in Fig. 72. The non-coalescence of the drops is still without explanation.

If a pith ball hung by a fine thread be brought up against the side of the jar, it is driven away at once, straight out if at H, I, K or L, Fig. 71, sideways if at any other point.

The existence of transverse motion in bell vibration at once explains a well-known mode of sounding a wine-glass or a finger-glass by drawing the wet finger round the edge. Disregarding the tangential part of the vibration, it would be difficult to see how the tangential motion of the finger could arouse the vibration at all. But it is easy to see that the finger produces a tangential displacement which must be accompanied by a radial displacement, just as the clapper of a bell produces a radial displacement which must be accompanied by a tangential displacement.

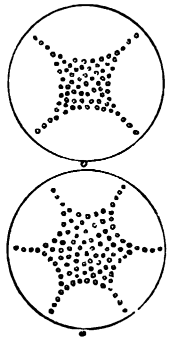

FIG. 72.—Drops on surface of Methylated Spirit in vibrating Glass Dish. O, point of bowing; 4 and 6 nodal points are shown.

Lord Rayleigh has shown that along the axis of a bell there is a great falling off in the sound. This is exactly what we should expect from the opposite motions of successive segments, which should neutralise each other's effects at a point equidistant from all. Probably if echoes could be excluded the axis would be a locus of silence.

Though the shape of a bell renders it impossible to calculate the frequencies of its vibrations by theory, we may, nevertheless, find the ratio of the frequencies of two geometrically similar bells by the Principle of Dynamical Similarity (see p. 119), for that principle gives

$$T_1 : T_2 = l_1/\sqrt{E_1/\rho_1} : l_2/\sqrt{E_2/\rho_2},$$

or

$$= l_1 : l_2,$$

if the bells are of the same material.

The Vibrations of Membranes.—These have been investigated both theoretically and practically. The theoretical investigation supposes the membrane to be perfectly flexible and uniformly stretched by a constant tension; in fact, the membrane is to the plate what the string is to the bar. It is only necessary here to say that the calculated results agree with experiment as to the positions of the nodal lines, but that the frequencies are not correct, owing probably to the influence of the air. The best experiments have been made by Bourget, partly in conjunction with Bernard.[1] Paper was found to make a good membrane. It was glued on to a frame while wet, and the shrinkage on drying produced the requisite uniform tension in some cases, though there were many failures. The membrane was set in vibration by resonance. A series of gradually ascending pipes was used, and when the frequency of the pipe sounded agreed with that of a mode of vibration of the membrane the latter responded, and its nodal lines were exhibited by sand, as in the case of plates.

Savart supposed that a membrane could, and did, respond to any sound, and that the transition from one set of nodal lines to another was gradual. No doubt there would be forced oscillations excited by any tone, but Bourget showed that only when the source was nearly in unison with one of the natural tones of the membrane were the vibrations sufficiently strong to form the nodal patterns. Savart used a pipe considerably higher than the gravest tone of the membrane, and, since the overtones crowd together as the frequency rises, the pipe was always near an overtone.

The modes of vibration have a considerable resemblance to those of plates. Thus in round membranes there may be either nodal circles—starting with a single circle round the boundary—or nodal diameters—starting with one—or combinations of circles and diameters.

Sedley Taylor has devised a very beautiful method of exhibiting the modes of vibrations of membranes, by means of soap film. A film is stretched across a frame and placed over an air cavity. When a note is conveyed into this cavity by a pipe the membrane vibrates, and the nodal lines, being of different thickness from the rest of the film, are shown by the interference colours. Some most beautiful effects are produced, and the colours may be exhibited by the lantern. The instrument is termed the Phoneidoscope.

The drum is an example of a membrane. The tension is supplied by the stretching frame, and the drum, when struck, gives out a note of its own.

The drum of the ear is another example of a membrane. It closes the external aural cavity, and is set in forced oscillation by the waves reaching it. No doubt, its tension is unconsciously varied, so that it will more easily take up the vibrations reaching it. It is probable that the restoring force is not proportional to the displacement, the system being complicated by the small chain of bones connecting the drum with the internal ear.

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[1] Rayleigh's *Sound*, i. § 213.

CHAPTER IX.

VIBRATIONS MAINTAINED BY HEAT—SENSITIVE FLAMES AND JETS—MUSICAL SAND.

CONTENTS.—Trevelyan's Rocking Bar—Singing Flames—Sounding Tube—Sensitive Flames and Water-Jets—Musical Sand.

CERTAIN phenomena now to be described are linked together by the fact that musical vibrations are maintained by the communication of heat. The simplest case is that of **Trevelyan's Rocking Bar.** This is a brass "bar" of the shape shown in Fig. 73.

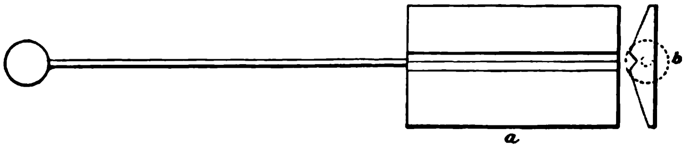

FIG. 73.—Trevelyan's Rocking Bar; face *a*, section *b*; about ¼ size.

When heated to some point below the melting-point of lead, and placed face downwards, with the edges of the V groove (shown in the section *b*) on the edge of a lead plate, the bar begins to "sing," emitting a very rough note. The note may be made to rise in pitch by slightly pressing the bar. We may explain the action thus:—

When one edge of the V groove comes in contact with the lead, it makes a little depression which tends to rise up again and throw the edge up; in other words, the edge rebounds. But the edge has given heat to the lead which makes it expand, and all the more since, for a metal, lead is not a very good conductor. The heating is therefore comparatively local. It takes place, also, chiefly during the rebound, since there is some lag owing to the time taken for heat to be conducted from the brass. Hence there will be an expansion of the lead upwards, aiding the rebound more than it neutralises the original fall of the bar. More work is done upon the edge during the rebound than was done by it during the depression, and energy is given to the bar. The other edge then falls down and goes through the same process, and there is a rapid rocking, the vibrations increasing until the supply of mechanical energy from the heat is equal to that radiated out as sound.

Singing Flames.—When a glass tube, say of $\frac{3}{4}$ inch or 1 inch internal diameter, is lowered over a small gas jet, to which the gas is supplied through a very fine hole at the top of a narrow tube, the air in the wider tube begins to sound its note as an open pipe. When the jet is about $\frac{1}{5}$ of the way up, if the conditions are favourable, the jet appears at once to lengthen out, and by carrying the eye rapidly past the jet, or by using a revolving mirror, the intermittent appearance shows that it is jumping up and down, probably, indeed, going out and being rekindled once for every vibration. Though the effect is obtained sufficiently well from common gas, it is still more striking from hydrogen. This may be generated in a flask, through the bung of which the jet tube passes, as in Fig. 74. A little experience with various lengths of jet tube and sounding tube shows very clearly that success depends chiefly on the relation between their lengths. If the lower end of the jet tube be open, as in Fig. 74, then its length must be less than half, or more than the whole length of the sounding tube. The explanation of this was first given by Lord Rayleigh,[1] and it is so excellent an illustration of the conditions under which vibration may be maintained, that we shall consider the subject at length.

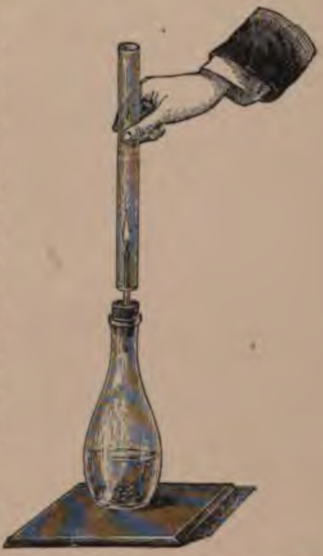
FIG. 74.—Hydrogen Singing Flame

It is clear that in this case the energy is supplied intermittently as heat by the successive kindlings of the flame. To understand at what phase of the vibration the supply must be given to maintain the vibrations, let us consider an abstract case, that of a column of air free from viscosity, and contained in a non-conducting vertical cylinder, closed at the lower end and having near the upper end a smoothly working weightless piston, with the atmospheric pressure always exerted on its upper side. Since the piston is weightless, the pressure on the under side is also atmospheric, for otherwise we should have infinite acceleration. Vibrations once started in the air column will be maintained, the air-column forming one quarter of a stationary wave from a node at the closed end to a loop at the piston.

Now the effect of a sudden communication of heat diffused uniformly, we shall suppose, through the whole mass, is to increase the temperature and the pressure at once, and the mean volume will also be increased. In other words, the centre of swing of the piston is removed farther from the end of the cylinder.

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[1] *Nature*, vol. xviii. 1878, p. 319; *Sound*, ii. § 322, *f–j*.

Bearing this in mind, we see that if the heat is communicated when the piston is at A (Fig. 75), the movement of the centre C outwards increases the amplitude, *i.e.* increases the extent of vibration. The time is affected only very slightly, for it equals $4l/U$, where l is the mean length of the column of air and U is the velocity of sound. Since l is proportional to the volume, and this, in turn, to the absolute temperature, when the pressure is constant, and since U is proportional to the square root of the absolute temperature, the period is evidently proportional to the square root of the absolute temperature.

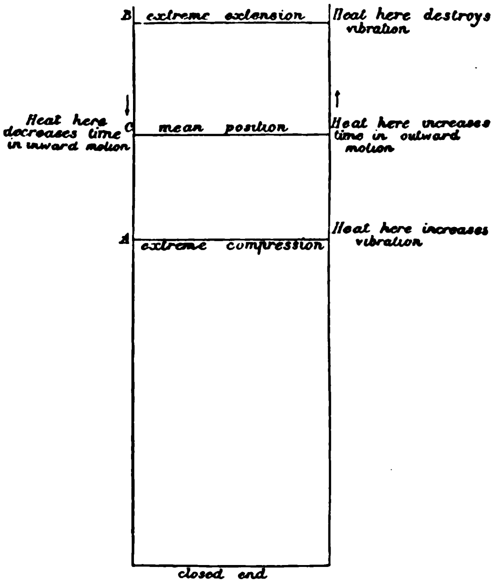
B
extreme extension
Heat here destroys vibration
Heat here decreases time in inward motion
C
mean position
Heat here increases time in outward motion
A
extreme compression
Heat here increases vibration
closed end

FIG. 75.—Influence of Heat Communication on the Vibrations of an Air Column.

If, on the other hand, the heat is communicated at B, the centre of swing, C, is suddenly brought nearer to B, the amplitude is decreased, and so the vibration is decreased. As before, the time is only slightly affected. If the heat is communicated at C, we must consider the direction of motion. If the piston is on its outward journey, then the centre of swing is suddenly moved forward in the outward direction, and the piston takes longer time to reach it; in other words, the periodic time is increased. The kinetic energy, however, remains nearly the same, for the restoring force near the centre of swing is small, and but little work will be done between the old and new positions of C. The piston starts from its new position with nearly the old velocity, and with nearly the same forces on the air, and so goes through

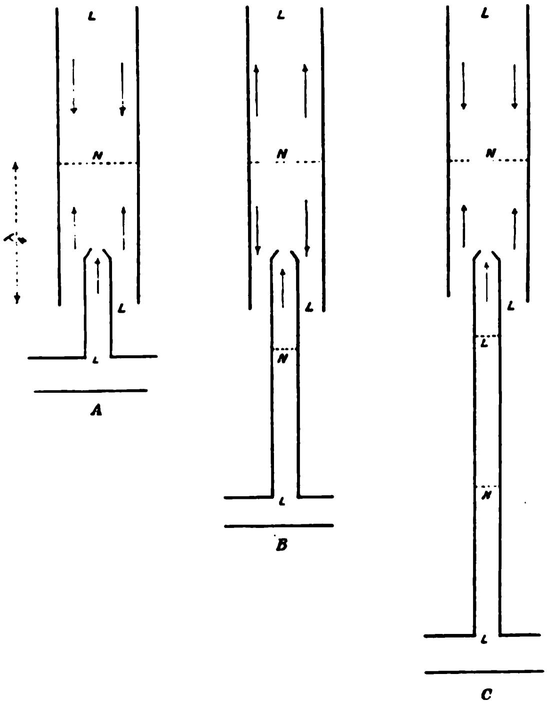

FIG. 76.—Singing Tube, with three different lengths of jet tube. The jet tube must have a loop below, where it joins on to a wider tube or gas reservoir. The arrows show the simultaneous motions necessary for equality of pressure in the two tubes at the jet orifice. A and C, vibrations maintained; B, vibrations destroyed.

nearly the same swing about the new position as it did about the old. Contrast this with the first case, in which extra work was done by the restoring force through an equal distance, but at the greatest elongation, where it added the most kinetic energy. Then in this case we conclude that the periodic time increases, but that the amplitude of vibration is nearly constant. If the piston is on its inward journey when the heat is given, the centre of swing moves out to meet it, and so shortens the time. As in the last case, the amplitude is but little affected.

If the heat is given, not at definite instants, but over some part of the time of vibration, there will be both change of amplitude and of period; but we may conclude, generally, that the communication in the half period during which the pressure is above the normal, increases the vibration, while during the extension half period it decreases the vibration. The effect on the time does not here concern us, but evidently during the inward motion heating increases, and during the outward motion it decreases the period.

Removing the impossible surroundings and conditions, and coming to the actual case of a sounding pipe, we see that if heat be communicated during compression, the vibration will increase until the energy lost by the production of sound-waves is equal to that gained in the mechanical form from the heat imparted.

Turning now to the singing flame, when the air within the tube is in a condition of vibration, it makes forced oscillations of the same period in the jet tube; the condition which the jet tube must satisfy being that of equality of pressure always at the orifice of the jet in both systems. We may suppose the jet tube to have an open end below, where it joins a wider tube or gas reservoir, so that this is necessarily a loop.

Let A, B, C (Fig. 76) represent different lengths of jet tube for a given length of sounding tube. In A, where the jet tube is less than $\frac{\lambda}{4}$, the pressure will be the same at the orifice in both tubes only if the motion in both is to or from the central node of the wide tube at the same time. Hence the gas will flow from the jet to the sounding tube, while the air is moving from maximum extension to maximum compression, and will be checked in the other half vibration. There will be a retardation of the heating, partly due to the time taken for the gas to burn, and partly due, perhaps, to the expulsion during the first part of the motion of incombustible air drawn in during the previous part of the vibration. Hence the heating is mainly thrown into the compression half of the vibration, and the vibration is maintained. In B, where the jet tube is more than $\frac{\lambda}{4}$ and less than $\frac{\lambda}{2}$ in length, there is a node in the jet tube. For equality of pressure at the orifice there must be motion to the nodes at the same time in both tubes, and motion from at the same time. Hence, at the orifice the motions in the two tubes are in opposite phases, and the gas is drawn into the sounding tube during the half vibration from maximum compression to maximum extension. Since there is a lag in combustion and heat-giving, the heating occurs mainly in the half vibration during which the extension occurs, and so the vibration tends to die away. When the tube is long enough to have two loops, as in C, the conditions of A recur, and the vibration is again maintained. If there is a node exactly at the orifice, there are merely variations of pressure, and the current of gas is not otherwise affected. The vibrations, therefore, are not interfered with.

Though long cylindrical tubes are generally employed as sounding tubes, lamp chimneys or globes may be used. It is easy, too, to obtain sounds from a tube closed by a bung at the upper end, if air be drawn through a hole in the bung in order to maintain combustion.

Rijke's Sounding Tube.—A very remarkable effect, discovered by Rijke, is obtained by inserting a piece of metal gauze about a fifth of the way up a long and wide tube and stretching across the tube. The tube is held vertical and the gauze is heated red-hot by a gas flame. On withdrawing the flame, a most powerful note issues while the gauze cools, so long as the tube is vertical, but ceases at once if the tube is turned into the horizontal position. The explanation is similar to that just given for the singing flame. There is an up-draught of air—the convection current due to the heated gauze—and superposed on this the vibrations of the air set going by any disturbance. The gauze communicates most heat during the condensation half of the vibration, and so the variations, if once sufficiently started, are maintained. The reason for the greatest heat-giving during condensation may perhaps be seen most easily from the time-displacement diagram, Fig. 77,

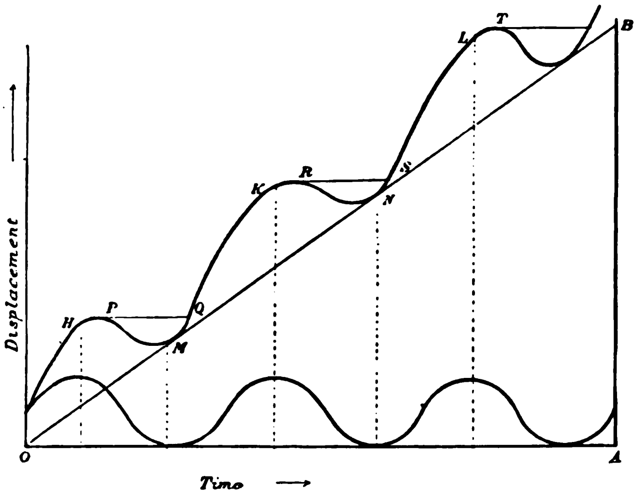

FIG. 77.—Time-Displacement Diagram for a layer of air near the gauze during several vibrations in Rijke's tube. QR, ST, fresh air drawn through the gauze and most heat given.

which represents the motion along the tube of a single layer of air in the neighbourhood of the gauze during several vibrations. The sine curve along the time line OA represents the displacement due to vibration alone, and the line OB the displacement due to the convection up-current alone. Their summation, HMKN . . . represents the actual displacement. If PQ and RS are horizontal tangents at P and R, the displacements from Q to R and from S to L represent a fresh advance over entirely new ground, so that in the corresponding times new cold air is coming against the heated gauze and the communication of heat is greatest. From P to Q and from R to S the displacements are over old ground. The air going through the gauze has already been through it, so that it is already warm and will not take so much new heat. Now, the points of maximum condensation are HKL, where the tangents are parallel to OB, and the points of maximum extension are MN, where the curve touches OB, so that QR and ST lie more in the half vibration with pressure greater than the normal than in the other half.

If the vibrations are so small that the curve in Fig. 77 ascends all the time, then fresh air is always passing through the gauze and the vibrations will no longer be maintained by the heat. Perhaps some such effect explains an observation made by Schaffgotsch and Tyndall, that when an ordinary singing tube with a jet in it is silent, it may be made to sing by sounding its note from another source. Increasing the amplitude of the vibration may throw the heating more decidedly into the proper part of the period.

Geyer's Singing Flame and Govi and Barrett's Sensitive Flame.—If a piece of metal gauze is held above a glass tube burner with orifice, say from $\frac{1}{10}$ to $\frac{1}{30}$ inch wide, and if the gas is lighted above the gauze, the flame is conical, quiet, and luminous when the gauze is just above the burner, and it is non-luminous, roaring, and spread out when the gauze is raised. There is a critical point just before roaring begins, at which the flame is exceedingly sensitive to certain ranges of sound. The narrower the orifice of the burner, the higher are the sounds to which the flame responds. When a suitable sound is made, the flame suddenly falls down for a moment and begins to roar, and if the adjustments are carefully made the sensitiveness is extraordinary.

Geyer found that if a tube, say 12 inches long and 2 inches in diameter, is placed over the flame when roaring, the tube sings loudly and continuously, far more loudly than the common singing tube. If the flame is in the sensitive state and the tube is placed over it, it only sings when a note is sounded of the kind disturbing the flame.

If, for instance, a narrow orifice of $\frac{1}{30}$ inch be used, the flame is sensitive to very high notes, such as are contained in the sound of the letter S. On speaking to the flame, it will pick out the S sounds and sing, or rather scream, for a moment every time the sound recurs.

The maintenance of the note of the singing tube placed over the roaring flame is probably to be thus explained. In the first place, it gives rise to disturbances producing vibrations large compared with the up-draught; and in the second place, the flame being much more spread, the velocity of the gas is less, and so it can be more easily checked and aided, both conditions necessary for maintenance.

The Singing Bulb.—Sometimes when a bulb has been blown at the end of a narrow tube it sings while cooling. Lord Rayleigh explains this by the unequal heating of the stem.

Suppose that a vibration has in any way been started. During the inrush from maximum extension to maximum condensation the air is being carried from cooler to hotter parts of the stem, and is therefore in the most favourable condition to receive heat. During the following outrush it is being carried from hotter to cooler parts, and is therefore in a less favourable condition to receive heat. Doubtless there is some lag in the phase of heat-giving, and so there is a balance of heat-giving in favour of the half of the vibration during which there is compression; thus the vibration is maintained. The continual in-draught as the bulb cools renders the case similar to that of Rijke's sounding tube and necessitates vibrations of a certain size before maintenance begins.

The Radiophone.—We pass now to some phenomena in which intermittent heating leads to a musical note of the frequency of the heating, the periodicity being already impressed on the heating.

Messrs. Graham-Bell and Tainter found that a regularly intermittent beam of light of sufficient frequency falling on a thin solid disc—almost any solid is effective—stretched across the end of an ear-trumpet gave rise to a musical note of the same frequency as the intermittence of the beam. The alternate heating and cooling of the front face of the disc produces alternate bulging and sinking in of its surface, and so air-waves are sent down the hearing tube. Bell proved that the disc had this to-and-fro motion by placing a short stout wire with a disc at the end in contact with the disc exposed to the radiation. The second disc then sounded, showing that waves were sent along the wire.

Subsequent researches showed that similar effects could be produced by allowing the intermittent beam to fall on cavities containing cotton-wool, worsted, silk, or, best of all, lamp-black. Liquids and gases in test-tubes were also active. The evident explanation of expansion and contraction, either of the material itself, or, when this is porous, of the contained air, is no doubt to be accepted. The discoverers of this kind of vibration have termed any device for showing it a Radiophone.

In one very interesting case the beam is rendered intermittent by the voice. On its way to a sensitive disc, on which it is converged as a focus, the light is reflected from a thin plane glass or mica mirror. If the plane mirror is spoken to, it curves under the vibration and alters the position of the focus, throwing it alternately in front of or behind the sounding material, and so altering the quantity of heat absorbed. In this way musical tones have been transmitted some distance, and with lamp-black even speech has been reproduced 40 metres from the transmitting mirror.[1]

Sensitive Flames.—If a jet of gas be allowed to pass through a circular orifice, such as that of a glass tube drawn out slowly and cut off with an opening $\frac{1}{16}$ inch in diameter, the flame burns with a long tapering form. On increasing the pressure, a point is reached at which it shortens and begins to roar. On reducing the pressure till the flame is just below the roaring point, it is exceedingly sensitive to a certain range of notes, dipping down and roaring for a moment every time one of these notes is sounded. Usually, with considerable pressure, high notes, such as those in a hiss or in the rattling of keys, are most effective.

Tyndall has studied the conditions of sensitiveness in great detail, and the reader is referred to his account.[2]

Various forms of flame are found to be sensitive, sometimes even those from common burners. One of the best is that of Govi and Barrett, already described in connection with Geyer's singing tube. Combustion is not an essential part of the phenomenon, for smoke jets are at least as much affected. The flame merely serves to show the form of the jet. If, for instance, hydrochloric acid and ammonia are mixed with a jet of air, the jet is at once visible, and the sensitiveness may be noticed as with a flame.

Lord Rayleigh has shown that if the effective note is made to form a series of stationary waves by reflection against a hard wall, the jets are affected at the loops and not at the nodes; in other words, variations in motion are effective, and not variations in pressure. It can be shown that the disturbance occurs in the orifice by directing the sound by a tube first to points above or below the orifice, when it is ineffectual, and then to the orifice, when the flame at once responds.

Lord Rayleigh[3] has investigated a theory of the instability of fluid jets, and has shown that for a certain range of velocity a given jet will be unstable for a given range of periodic disturbances, *i.e.*, the disturbances, once made, will go on increasing. The theory thus accounts generally for the observed facts, but the details still require working out. In the sensitive flame the motion appears, from experiments made by Lord Rayleigh, to be converted by the disturbance from a more or less straight into a sinuous outrush.

Sensitive Water-Jets.—When a jet of water issues through a circular hole, say $\frac{1}{10}$ inch in diameter, from a cistern two or three feet above the hole, it is sensitive to vibrations. Ordinarily a jet issuing from a round hole is at first a smooth cylinder, but at some little distance from the hole it breaks into drops. These go at different rates, and therefore collide, rebound, and scatter. But if the orifice be put in solid communication with, say, a tuning-fork of suitable pitch, the jet appears to be continuous, and there is no scattering. Really the drops are formed as before, but now at quite regular intervals, so that they follow in orderly procession. If the jet be illuminated by an intermittent beam of the same frequency as the influencing vibration, the regular issue of drops is revealed; for, as one drop exactly moves into the position of the next between two successive illuminations, they all appear at rest. If the periodic time of the light is rather greater, the drops move rather more than the space between during the darkness. Each appears to have moved a little farther on, and there is a slow procession. If the periodic time of the light is rather less, there appears to be a slow retrogression.

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[1] *Nature*, xxiv. p. 42.

[2] Tyndall, *Sound*, Lecture VI.

[3] *Sound*, 2nd ed., Chap. XXI.

The regular separation of the jet depends upon the fact that a cylindrical surface of liquid, possessing a uniform surface tension, is unstable if its length is more than π times its diameter. This may be illustrated by stretching a soap film between two equal circles, say between a ring and the end of a pipe, through which air can be blown to maintain the internal pressure. On drawing the ring from the pipe, and keeping up the air pressure so as to maintain the cylindrical form, as soon as the length slightly exceeds three times the diameter, the film pinches in at one end or the other and divides into two, one across each ring. The instability is the greatest if the length is half as great again as this. As a stream of water issues from an orifice, the cylindrical part is more than π times as long as it is wide, and so its surface is unstable. If a tremor at the orifice makes a depression on the surface of the jet, this is carried forward and tends to deepen as it goes, through the instability. The velocity, however, carries the jet forward some little distance before complete severance occurs, and the continuous part is therefore far larger than three diameters. If the depressions are made quite regularly by a tuning-fork in connection with the orifice, equal-spaced depressions or necks are impressed on the jet, and it breaks up regularly when these are farther apart than about three diameters. If about 4½ diameters apart, the instability is greatest and the cleavage is most rapid.

Assuming, for the purpose of illustration, that a jet has diameter d and velocity v, due to a head h, so that $v^2=2gh$; then for maximum effectiveness—

$$\frac{9}{2}dn=v=\sqrt{2gh}$$

where n is the frequency of the fork, and

$$n=\sqrt{\frac{8gh}{81d^2}}.$$

If $d=\frac{1}{100}$ foot and $h=4$, $n=356$ nearly.

The orderly procession of drops may be easily shown in another way, viz., by allowing the stream to fall on a sheet of stiff paper, or perhaps better, on a tin canister provided with a tube leading to the ear. The note of the disturbing fork is clearly heard as due to the patter of the drops.

Mr. Chichester Bell has devised some very striking experiments in which this patter is rendered audible. He finds that a vertical jet descending from a very fine hole, say $\frac{1}{75}$ inch in diameter, in a glass tube under a pressure of about 15 feet of water, is peculiarly sensitive. When it is allowed to impinge on a sheet of india-rubber stretched over a tube half an inch in diameter, sounds of suitable frequency communicated to the supply tube are enormously magnified by the impact on the india-rubber. The ticking of a watch may be rendered audible in this way to a large audience by placing it on a piece of wood touching the tube. Evidently the vibrations at the orifice are imprinted on the jet, and the impressions deepen through the instability. If the stretched sheet is struck by the jet before it has actually broken up, the rate at which it receives momentum, or the pressure on it, is proportional to the cross section of the jet, which is variable with the vibrations. The pressure variation is greater the farther the sheet is from the orifice, and therefore the greater is the amplitude of the vibrations of the sheet.

Some very interesting experiments on water-jets are described in Boys's *Soap Bubbles*. An account of the method of producing Chichester Bell's jets is also given there (p. 161).

Musical Sand.—Near Mount Sinai there is a place termed El Nakous (*nakous* is the name for a metal plate used in Greek convents in the East in place of a bell), where are two sandy slopes so steep that any disturbance of the sand causes it to slide down. The heat of the sun even is sufficient to set it in motion. As it slides it emits a musical note. According to an account[1] by a Mr. Seetzen, who visited the place some time before 1812, the sound appeared "to have the greatest analogy to the humming-top; it rose and fell like the sound of the Æolian harp."

Probably the phenomenon is to be classed with the curious musical notes emitted by many sands when sharply struck. These sands are found in various parts of the world, the most remarkable examples in our islands occurring in Eigg, and at Studland Bay, near Swanage. At Barmouth, in North Wales, there is another example on the shore on the river-side opposite the harbour.

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[1] Herschel's *Sound*, § 380.

Mr. Carus Wilson[1] says with regard to the Studland Bay sand: "The best results were obtained by drawing a thick deal rod, on to the end of which I had fixed a resonator, over the surface of the sand. Sounds produced in this way were heard unmistakably for a considerable distance. The patch averaged 7½ yards in width, and ran parallel with the trend of the shore for some hundreds of yards. The sand on the sea side of the patch was fine and emitted notes of a high pitch; that on the land side was coarse and emitted notes of a lower pitch. The rod drawn across the patch, therefore, gave a great variety of pitch." In investigating the conditions for sounding, he found that the chief one was that the sand should consist of equal clean rounded grains of quartz, and he even succeeded in making a hitherto silent sand musical by carefully sifting out round grains of equal size and boiling them in dilute hydrochloric acid to cleanse their surfaces. Sands which were at first musical became silent if often struck, probably through clogging with the abraded particles. Mr. Wilson has found that the musical quality may be sometimes restored by sifting, washing, and boiling. He has observed that the most musical notes are obtained when the sand struck is contained in a vessel with hard polished sides, such as a teacup. A rough interior renders most sands mute. An important paper by Prof. Osborne Reynolds, "On the dilatancy of media composed of rigid particles in contact,"[2] may give the key to the explanation of these phenomena, which is possibly to be found in the fact that there is an arrangement of minimum volume for a number of equal spheres in contact. We may suppose the sand to consist of equal spheres, arranged, when undisturbed, so as to occupy minimum volume. When disturbed the mass may pass through many successive minima of volume before coming to rest, and if we can suppose that the time occupied in passing from one minimum to the next is constant, a musical note should issue. The smoothness of the containing vessel should enable the outer layer of particles more easily to change their position, and so more easily to assume successive minimum volume arrangements.

Probably the squeaking of frozen and powdery snow, when trodden on, is to be classed with the sounds obtained from sand.

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[1] *Nature*, vol. xliv. 1891, p. 322.

[2] *Phil. Mag.*, xx. p. 469.

CHAPTER X.

THE SUPERPOSITION OF WAVES.

CONTENTS.—Interference of Sound-Waves— Beats—Concord and Discord—Combination Tones.

Interference of Sound-Waves.—When two trains of waves of the same length arrive at the same point, their superposition produces a disturbance of the frequency of either, and of amplitude depending on their relative phases. If the amplitudes are equal and the phases the same, the superposition gives a double vibration with an average double velocity, and, therefore, with four times the energy. If the phases are opposite, then union of the trains produces no vibration, and there is silence. With any intermediate relation of phases there will be a vibration of intermediate amplitude.

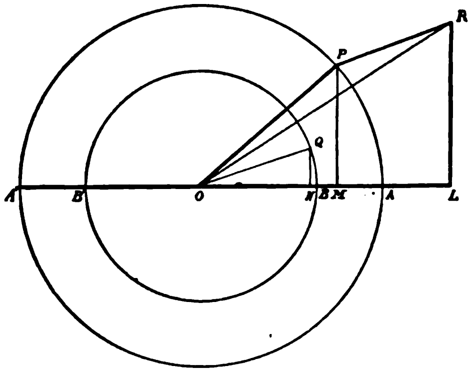

FIG. 78.—Resultant of two Harmonic Vibrations of the same frequency but different phases. OM, ON, harmonic displacements corresponding to the circular motions of OP and OQ; OR, the vector sum of OP and OQ; OL, the resultant harmonic displacement.

The general result of the superposition of vibrations of the same frequency may be easily obtained from the trigonometrical expression for the sum of the disturbances (see p. 66); but the following geometrical method is perhaps preferable, and is of value also in the case where the two vibrations have different frequencies.

Let OA represent the amplitude of one vibration, OB that of the other (Fig. 78). Then, drawing circles on AOA′ and BOB′ as diameters, and allowing radii OP, OQ, to revolve uniformly round these circles, the points MN at the feet of the ordinates PM and QN may be taken as representing the vibrations. The angles POA, QOB, represent the two phases, and POQ, which is constant, represents the difference of phase. The resultant displacement is OM+ON. Now draw PR parallel and equal to OQ, and therefore making a constant angle with OP. The projection OL of OR on OA equals the sum of the projections of OP and PR, or of OP and OQ; *i.e.* equals OM+ON. Hence, making the triangle OPR revolve uniformly round O, the projection of R gives the resultant vibration. If the amplitudes of the components are a and b and the difference of phase a, the amplitude c of the resultant is

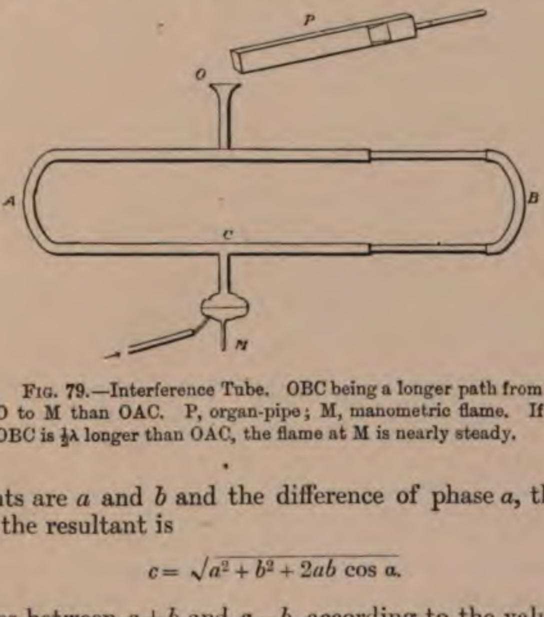

FIG. 79.—Interference Tube. OBC being a longer path from O to M than OAC. P, organ-pipe; M, manometric flame. If OBC is $\frac{1}{2}\lambda$ longer than OAC, the flame at M is nearly steady.

$$c = \sqrt{a^2 + b^2 + 2ab \cos \alpha}.$$

This varies between $a+b$ and $a-b$, according to the value of a.

Such superposition of vibrations is termed **interference**, and many illustrations may be given of its occurrence with sound-waves. If a Y tube is held with one prong over each of two adjacent segments of a vibrating plate, and the third tube is connected to the ear, comparatively little sound is heard, since the waves coming up the two prongs are in opposite phases. If the two are held over the same segment, the phases are the same, and the sound is far louder.

In Fig. 79 is represented another mode of exhibiting interference. M is a manometric flame, to which the sound-waves from a source, say an organ-pipe, can be carried by two channels, OAC, OBC. The length of OBC is adjustable, and when it is pulled out $\frac{\lambda}{4}$, the path difference is $\frac{\lambda}{2}$. The waves, therefore, arrive at M in opposite phases and neutralise each other, and the flame is nearly steady. With a shorter or longer path difference the flame is disturbed, and a revolving mirror near M may be used to show how it is affected.

When a tuning-fork on its box is sounded near a wall, the direct and reflected train of waves are superposed in the space in front of the wall. If, for a given point, the path difference is $\frac{\lambda}{2}$, the two trains nearly neutralise each other, and the sound is a minimum at that point. If the difference is 0 or λ, they reinforce each other, and the sound is a maximum. As the fork is moved out from the wall a series of maxima and minima are heard as the path difference passes from the value 0 through the values $\frac{\lambda}{2}$, λ, $\frac{3\lambda}{2}$, &c. This may also be regarded from another point of view. The reflected waves may be supposed to issue from the reflection of the fork in the wall, considered as a mirror. One source is moving towards the point, the other away from it, and the frequencies of the two trains will be altered in opposite ways, according to Döppler's principle. Beats will, therefore, occur between the two notes.

The stationary waves formed by the direct and reflected trains *between* the source and the wall may also be regarded as examples of interference, the nodes occurring where the path difference is 0, λ, 2λ, and the loops or the double motion where it is $\frac{\lambda}{2}$, $\frac{3\lambda}{2}$, &c.; for now the opposite directions of motion will have the same effect as a reversal of phase throughout one train.

The sensitive flame has been applied in this case by Lord Rayleigh to measure the wave-lengths of sources of frequency high enough to affect the flame. The source should be a bird-call or very high-pitched whistle of nearly pure tone. If this is blown steadily a few feet in front of a large board, a series of stationary waves is formed, with the first node at the wall. The burner of the sensitive flame is then moved to and fro in the line from the source perpendicular to the wall, and the flame is least affected at the nodes. The average distance between successive positions of least effect is half a wave-length. If a tube connected to the ear is used, the points of silence, on the other hand, are at the loops.[1]

Other interference experiments have been devised corresponding to the bi-prism, to the Huygens zones and to the circular disc experiments in light.[2]

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[1] *Phil. Mag.*, vii. 1879, p. 153.

[2] Rayleigh's *Sound*, ii. § 292*a*.

Beats.—If two trains of waves, which differ slightly in frequency, reach the ear at the same time, we do not usually perceive the two tones, but a single tone having the well-known variation in intensity which is described as "beating." This is due to the fact that at certain equal intervals the trains of waves agree in phase and reinforce each other, while midway between they are opposite in phase and tend to neutralise each other. These points of maximum and minimum vibration travel on with the velocity of the sound to the observer and produce the beats.

Suppose, for instance, that two equal trains of frequency, respectively 256 and 260, are superposed, and that at a certain point they agree in phase. At a distance of 64 wave-lengths of one and 65 of the other they will agree again, while midway between they will destroy each other. At a distance of 128 and 130 wave-lengths respectively from the first point they will agree again, and midway between this and the last point of agreement they will destroy each other. The sound takes $\frac{256}{64} = \frac{1}{4}$ sec. to travel the distance between the points of agreement, so that every $\frac{1}{4}$ sec. the observer notes a swelling out of the sound, or there will be four beats per second. And, generally, if the frequencies are m and $m + n$, there will be n agreements in the train occupying the distance traversed by the sound in one second or n beats. The result of superposing two trains with frequencies in the ratio $8:9$ is shown in Fig. 80, taken from Müller Pouillet's *Physik*.

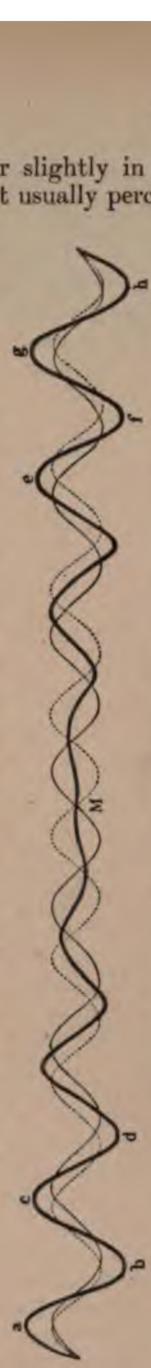

Fig. 80.—Superposition of Waves of Frequencies 8 and 9 respectively. The thin lines represent the components, the thick line the result of their superposition.

If the two trains have equal amplitudes a and frequencies m and $m + n$, the sum of the displacements at a given point may be represented by

$$a \sin 2\pi mt + a \sin 2\pi(m+n)t$$

$$= 2a \cos \frac{2\pi nt}{2} \sin 2\pi(m + \frac{n}{2})t.$$

This may be taken to represent a single tone of frequency $m+\frac{n}{2}$ midway between the frequencies of the two trains and of amplitude $2a \cos \frac{2\pi nt}{2}$. This amplitude varies from $2a$ through 0 to $-2a$, so that the intensity goes through its range of values in time $t = 1/n$ or the frequency of the beats is n.

This is in agreement with observation. A hearer does not perceive the tones separately, but a single tone of varying intensity midway between the two in frequency.

The graphical mode of representing two superposed vibrations in Fig. 78 also shows at once the number of beats per second. For if we suppose that N has greater frequency than M, OQ revolves more rapidly than OP. Every time that OQ gains a complete revolution on OP (or N a complete vibration on M), OPR will be a straight line and L will have a maximum amplitude. Midway, OQ will have gained half a revolution, and again OPR will be a straight line, but with R between O and P, and L will have a minimum amplitude. Thus, if the frequencies are m and $m + n$, OQ will gain n complete revolutions, and there will be n beats per second.

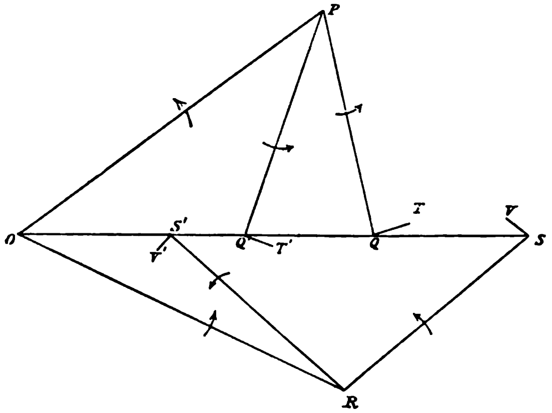

FIG. 81.—Composition of two unequal Vibrations of slightly Different Periods. The projection of the revolving radius OP represents the lower tone, that of PQ the upper. QT is the arc by which PQ gains on OP in one revolution of OP. If T is above OS, the resultant vibration time is less than that of OP; if below more.

The same mode of construction may be used to explain the fact that when two tones of nearly the same frequency are sounded the pitch of the resultant tone varies, as well as the amplitude, *if the amplitudes of the components are unequal.*[1] Let us consider the case in which the lower tone has the greater amplitude. Let OP (Fig. 81), revolving round O, represent the vibration of this lower tone in the manner of Fig. 78, and let PQ represent the higher tone, PQ being less than OP. If OP revolves counter clockwise, PQ revolves in the same direction, but rather faster, moving through the additional angle QPT, say, where QT is perpendicular to PQ, in the period of OP. If, then, at the beginning of a vibration, when the particle considered has its greatest displacement during that vibration, the two radii are in the position OP, PQ, when OP has completed its revolution, PQ has moved to PT; and OT being the sum of the two, the projection of T on OS now gives the vibrating point. But T is past OS, or the point has rather more than completed its vibration. Hence its period is shorter and its frequency is greater than that of OP.

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[1] Helmholtz, *Sensations of Tone*, 1875, App. XIV.; Sedley Taylor, *Phil. Mag.*, xliv., 1872, p. 56.

Similarly, when the two radii are in the position OR, RS, the frequency is again greater than that of OP, since SV = QT is on the upper side of OS.

When, however, the second radius is in either of the positions PQ′ or RS′, the frequency is less than that of OP; for, taking the first of these positions, by the time that OP has moved once round, PQ′ has moved once round and through the additional angle Q′PT′, and the projection of T′ now gives the vibrating point. But T′ is below OS, *i.e.* the point has not yet completed its vibration and the frequency is less than that of OP. Similarly with ORS′.

Returning to the first case, that of OPQ, we can easily see, by considering Q fixed and QP and PO revolving, that the frequency is less than that of QP.

Now the positions of OPQ, ORS, correspond to larger amplitudes, while the positions OPQ′, ORS′, correspond to smaller amplitudes. Hence, while the sound is louder, the tone lies between the two components, and while it is fainter the tone lies outside the lower, that is, is flatter than either.

It is not difficult to show that in the case of equal amplitudes the angle subtended by QT at O is always the same, so that the frequency is always the same and the mean of the two frequencies. Perhaps it is sufficient to note that if we first suppose the system to revolve round O and then round Q in Fig. 81, the angle of advance represented by QT in the one case will be equal to the angle of retardation in the other, so that the period must be midway between those of the constituents.

Concord and Discord.—When two pure tones are sounded together, and the pitch of one is gradually altered from unison with the other, the beats, of course, gradually become more frequent. At first they are not necessarily disagreeable. Indeed, in the organ-stop voix celeste they are purposely made by having two pipes slightly out of unison to each key. But as the interval between the two tones increases, the beat becomes more like a rattle or a jar, and the sound produces a disagreeable sensation. Even after the beats become too rapid to be separately perceptible, we are conscious of a roughness in the sound. This roughness we term discord or dissonance, and there is no doubt that it is the sensation corresponding to the rapid intermission which, when slower, we term beating. As the interval is still further increased there is a point of maximum roughness; after that, the dissonance diminishes, and finally ceases. Suppose, for example, that the two tones are, to begin with, $c''=512$. As we lower one, the roughness increases till the lower tone is about $b'=480$, making 32 beats per second with c'', and at an interval of a semitone with it. Beyond this the roughness diminishes, and about $a'=426\frac{2}{3}$, say 427, making 85 beats per second with c'' and at an interval of a minor third, it finally ceases. But neither the interval nor the number of beats for maximum or limiting dissonance is constant; for, as we rise in the scale, the number of beats increases, but not in proportion to the frequency, so that the interval diminishes. Fig. 82 is intended to exhibit this fact, the black lines showing constant intervals, while the dotted lines show the general nature of the intervals of maximum dissonance and limit of dissonance, without any pretence to exactness. A number of beats which may be very harsh between two high notes may hardly annoy us between two low notes. On the other hand, an interval which annoys us between two low notes ceases to be harsh between two high notes. The closing in of the interval of dissonance as the pitch rises probably accounts for the fact that it is easier to time high notes.

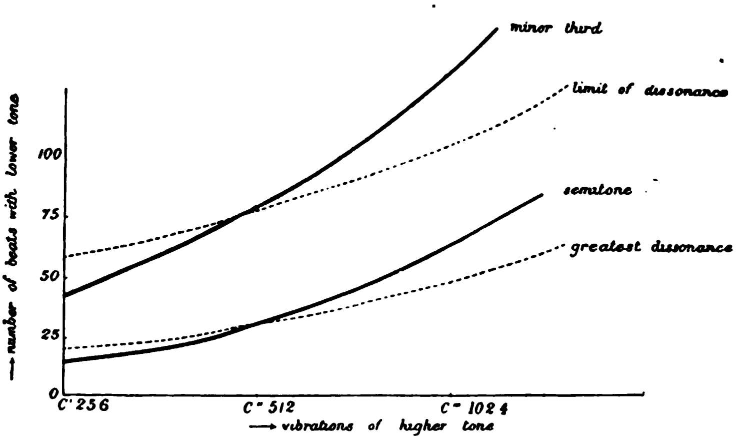


FIG. 82.—Diagram illustrating the decrease of interval, but increase of beats for maximum dissonance as the pitch rises; ordinates, number of beats, abscissæ, positions of the higher of the two tones on the scale.

Beats Between Harmonics or Overtones.—Undoubtedly dissonance occurs between notes played on ordinary instruments at an interval far beyond that which we have just accepted as the limiting interval, and averaging about a minor third. But we have supposed the notes pure tones, whereas almost all musical notes are really compound, with a number of overtones, usually of the harmonic series. Even when the two fundamentals are far beyond the dissonance range, some of their partials may come within that range and give rise to perceptible roughness.

Indeed, were all the overtones of the harmonic series present to a high order and all in equal strength, a single note would be discordant through beats between its own harmonics.

Putting down the frequencies of the harmonics and the intervals between successive pairs, as in the table below, it is evident that after the sixth the intervals are less than a minor third, and therefore the tones will beat with disagreeable frequency. Instruments are usually so constructed and excited that the higher partials are very weak or altogether absent. The effect of their presence may be realised by plucking a piano-string near one end, when the higher harmonics are prominent and the note is disagreeably rough.

Number of Harmonic and Frequency Relative to the Fundamental.	Interval between Successive Pairs.
1	
	Octave.
2	
	Fifth.
3	
	Fourth.
4	
	Major Third.
5	
	Minor Third.
6	

In briefly discussing some of the musical intervals, we shall stop short at the sixth harmonic, assuming that the higher ones are too weak to produce any noticeable effect, but that all are present up to the sixth.

Beginning with the octave, let us represent the two fundamentals by minims and their harmonics by crotchets as in Fig. 83. The upper note introduces no fresh overtones, and only changes the relative intensity of those already present in the lower. Hence, probably, the sense of repetition which we have with regard to the octave. Hence, also, its smoothness when exactly in tune. But if slightly out of tune there will be beating all along the series, and so the ear requires, and easily judges of, very accurate tuning.

Taking next the fifth (Fig. 84), only the third harmonic of the higher is not included in those of the lower note, but this being within dissonance range of the fourth and fifth of the lower, there is some roughness about the interval. If the interval is slightly out of tune, the second and fourth harmonics of the upper will beat with the third and sixth of the lower, and with some intensity, as the second and third harmonics are usually prominent. At the same time the upper third harmonic is brought nearer to the lower fifth, and so the dissonance is increased. Hence the fifth is an interval which is not when in tune so smooth as the octave, and easily becomes very rough when slightly out of tune.

The fourth, the major third, and the major sixth are represented in Fig. 85 with details of their dissonance and the modes of

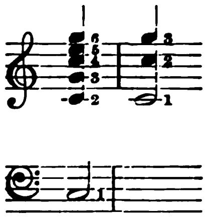


Fig. 83.—Octave with the Harmonics.

All the tones of the upper note already exist in the lower. Tuning secured by disappearance of beats in all these coincidences.

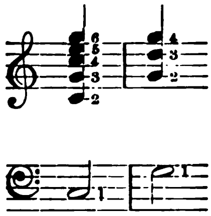


Fig. 84.—Fifth with the Harmonics.

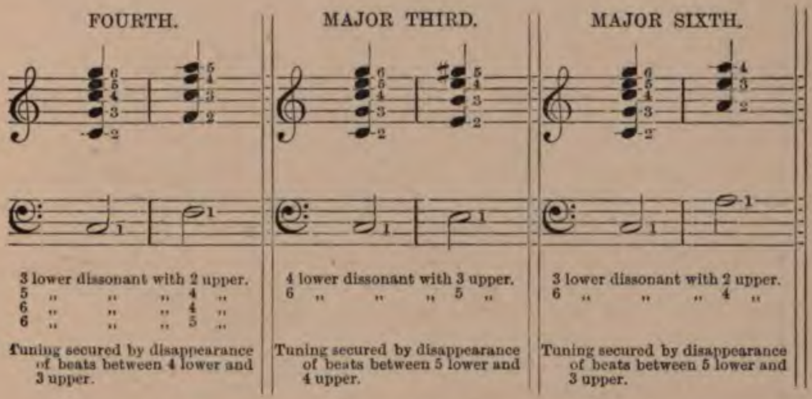
The third of the upper note is not in the lower, and beats with the fourth and fifth of the lower, giving some roughness. Tuning secured by disappearance of beats between the upper second and lower third.

tuning. Since the partials usually decrease in intensity as their order rises, it is evidently easier to tune the fifth than the fourth, and the fourth than the third or sixth.

Of course the nature of the instrument greatly affects the concords. For example, with stopped pipes the even harmonics are absent, and the interval of a fourth consequently gains in smoothness.

We have here only briefly indicated the principles of this physical aspect of concord and discord. For a full discussion of the subject the reader is referred to Helmholtz's great work, *Sensations of Tone.* A very good account will be found in Sedley Taylor's *Sound and Music.*

Combination Tones.—When two notes, each *separately* pure and free from overtones, such as those given by forks over resonance cavities, are sounded together, dissonance still occurs at quite wide intervals. Thus an octave of two separately pure tones is sensibly rough if loud enough and out of tune, and even beats may be heard as the tuning approaches exactness. The explanation is to be sought in the existence of **Combination Tones**, or tones existing in neither note separately, and only arising when they are sounded together. The most easily recognised combination tone is one having a frequency equal to the difference of the frequency of the two original tones or primaries. It may be heard when two organ-pipes or flageolets are blown hard close to the listener, especially if his attention is previously directed to the pitch of the tone he is to listen for. It is very noticeable, too, when a double whistle is blown or when two gongs are struck near together. The fact that the frequency of this tone is equal to the frequency of the beats formerly led to the supposition that the beats themselves merge into a musical tone. Thus Young[1] says: "When the beats of two sounds are too frequent to be heard as distinct augmentations of their force, they have the same effect as any other impulses which recur in regular succession and produce a regular note which has been denominated a grave harmonic." Now, a tone is ordinarily due to a succession of waves, such that the air in the first half of each wave is displaced forwards and the pressure is in excess of the normal, while in the second half the air is displaced backwards and the pressure is in defect. But



FOURTH.

3 lower dissonant with 2 upper.
5 " " " 4 "
6 " " " 4 "
6 " " " 5 "

Tuning secured by disappearance of beats between 4 lower and 3 upper.

MAJOR THIRD.

4 lower dissonant with 3 upper.
6 " " " 5 "

Tuning secured by disappearance of beats between 5 lower and 4 upper.

MAJOR SIXTH.

3 lower dissonant with 2 upper.
6 " " " 4 "

Tuning secured by disappearance of beats between 5 lower and 3 upper.

FIG. 85.—Intervals of Fourth, Major Third and Major Sixth.

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[1] *Lectures*, xxiii. p. 391, ed. 1807.

each half of a beat consists of a number of alternations of displacement of the air and excesses and defects of pressure, and in each half of the beat the mean displacement of the air is zero and the mean pressure normal. There is no reason to suppose that mere loudness produces a pressure of the drum of the ear inwards, while faintness allows a relaxation and return outwards; suppositions which would appear to be necessary if beats are to produce in the ear a tone of their own frequency.

An entirely different explanation of this and other combination tones, requiring no such supposition with regard to beats, has been given by Helmholtz.

This explanation depends on the fact that, with an unsymmetrical membrane, such as the drum of the ear, or with a limited air cavity subject to large vibrations, the restoring force is not proportional to the displacement. As we shall show below, it results that a simple harmonic force acting on the membrane or on the cavity will produce not merely a simple harmonic vibration of its own frequency, but a series of harmonic vibrations giving rise to overtones, which we may term **Self-Combination Tones.** When two harmonic forces of different frequencies act on the membrane or cavity, they produce vibrations having frequencies equal to the differences of frequencies of the original vibrations or their harmonics. These give rise to **Difference Combination Tones,** the first of which we have already described. There are also vibrations having frequencies equal to the sums of frequencies of the original vibrations or their harmonics. These give rise to **Summation Combination Tones.** The first summation tone may be heard with the harmonium, but not easily with other musical instruments.

Two pure tones, therefore, of frequencies m and n, entering the ear, may give rise to any or all of the following tones:—

m, n	primaries;
$2m, 2n$	self-combination tones;
$m-n$	first difference tone;
$m+n$	first summation tone;
$2m-n$	summation tone of first difference and one primary;
$m-2n$	difference tone of first difference and the other primary;

and so on, the various upper tones evidently being obtainable in different ways. The lower combination tones are the most important, but they all rise in importance as the original vibrations increase, and more than in proportion; for with the increase in amplitude of the vibration of the tympanum, the part of the restoring force not proportional to the amplitude comes more and more into prominence.

It is easily shown that these tones are usually formed in the ear of the percipient, and have no appreciable external existence, by using resonators. Did a given combination tone exist outside, an appropriate resonator held to the ear should reinforce it; but it is found that the resonator does not, in general, augment the sound.

In certain instruments, however, such as the harmonium, or siren as used by Helmholtz, the sound-waves issue immediately on their production from or into a confined air space, where the vibrations are so great that the restoring force is no longer proportional to the displacement, and combination tones result having an existence in the external air. These are more easily heard by the aid of resonators.

Rücker and Edser[1] have made an experiment directly proving the existence of combination tones in the air, the ear not being used at all. The sound from a double siren was directed on to a fork of frequency 64, the fork carrying on one prong a mirror forming part of a Michelson interferometer. When two notes were simultaneously sounded which should have a combination tone of frequency 64, the interference bands disappeared, but reappeared when the frequency was slightly different, thus showing that the fork was thrown into vibration by resonance to external vibration of its own frequency.

Even in free air combination tones are doubtless produced, since the force is not exactly proportional to the displacement, but they are quite negligibly small, and need not be taken into account.

To illustrate the effect of combination tones on consonance, it will be sufficient here to take the single case of the octave. For a fuller discussion we refer the reader to *Sensations of Tone* or S. Taylor's *Sound and Music*. Suppose that we have $c'=256$ and $c''=512$ sounding together. The first difference tone is 256 merging with one of the primaries. The first summation tone is 768, the third harmonic of the lower; and all the other combination tones will fall into the harmonic series. If the second primary is out of tune, flat, say, by 32 vibrations, the beating of the primaries is 224, being outside the dissonance range. But the first difference tone has frequency 224, equal to that of the beats, and so comes within dissonance range of the lower primary and a roughness is perceived.

General Mechanical Explanation of Combination Tones.[2]—Let us take the case of two reeds of frequencies m and n, mounted on a limited air-box and blown by air having an average excess of pressure P. First, suppose that one reed alone is vibrating with amplitude a and that the pressure excess is constant, then we may represent the issuing disturbance by—

$$Pa \sin 2\pi mt,$$

---

[1] *Phil. Mag.*, xxxix. 1895, p. 341.

[2] *Sensations of Tone*, 1875, App. XVI.

taking it as proportional to the pressure excess and to the amplitude of the reed. But the mere fact that the reed is vibrating affects the pressure excess with a periodicity, and we may, at any rate with nearer exactness, represent it by—

$$P\{1 + b \sin (2\pi mt + \epsilon)\}.$$

The issuing vibration is, therefore, to be taken as

$$Pa \sin 2\pi mt\{1 + b \sin (2\pi mt + \epsilon)\}$$

$$= Pa \sin 2\pi mt - \frac{Pab}{2} \cos (4\pi mt + \epsilon),$$

$$+ \frac{Pab \cos \epsilon}{2}.$$

The octave is present, therefore, as well as the fundamental tone.

Correcting the pressure by introducing a term corresponding to this octave, we should obtain the next harmonic, and so on, giving the series which we have called self-combination tones.

Suppose now that the second reed is sounding at the same time. Neglecting the variation in the pressure due to the first reed, we may put that due to the second as—

$$P\{1 + c \sin (2\pi nt + \eta)\}.$$

The issuing vibration will therefore be

$$Pa \sin 2\pi mt\{1 + c \sin (2\pi nt + \eta)\}$$

$$= Pa \sin 2\pi mt + \frac{Pac}{2} \cos \{2\pi(m - n)t + \eta\},$$

$$- \frac{Pac}{2} \cos\{2\pi(m + n)t + \eta\},$$

which gives the first difference and first summation tones. Recorrecting the pressure for these tones, we should obtain tones of the second order, and so on.

This investigation, of course, does not pretend to completeness, but it serves to give a correct general idea of the mode in which the combination tones are produced.

We may also give a rough general account of their production in the ear. Let us suppose that a single powerful tone reaches the ear. We may regard the alternations of pressure which it produces against the drum as giving a strictly harmonic force. Now if the vibration of the drum were also strictly harmonic, the total force, external pressure + internal restoring force, must also be harmonic. Since the external pressure is harmonic, this would imply that the internal restoring force is harmonic, and therefore proportional to the displacement. But the want of symmetry of the drum and its mode of attachment to the ossicles renders this impossible. In other words, the displacement due to a simple harmonic external force cannot itself be a simple harmonic. In fact, a term in the force proportional to the *square* of the displacement becomes more important as the displacement increases, and probably this tends to flatten the curve in the time-displacement diagram. The displacement curve, though not simply harmonic, is nevertheless truly periodic in the time of the external vibration, and may therefore, by Fourier's theorem, be resolved into a series of strictly simple harmonics, having periods once, twice, thrice, &c., the period of the fundamental. Thus we get the self-combination tones. The first of these, the octave, is sometimes heard when a tuning-fork, very sharply struck, is held over a cavity so as to give a very loud note.

When two notes reach the ear, since the elasticity or the restoring force per unit displacement is not constant, but varies with the displacement, each note has, as it were, to make use of an elasticity rendered periodic by the other note; and so we may expect to find an effect similar to that with two reeds, where each is blown by a pressure rendered periodic by the other. That is, we may expect to find summation and difference combination tones in addition to the self-combination tones.

Our knowledge of the physical concomitants of concord and discord, of which this chapter contains a very brief account, is almost entirely due to Helmholtz. His conclusions are now almost universally accepted, and even the experimental evidence urged against them seems, on further examination, only more fully to confirm them.

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